On the Scalability of Multi-objective Metaheuristics for the Software Scheduling Problem

Francisco Luna, David L. González-Álvarez, Francisco Chicano, Miguel A. Vega-Rodríguez
Introduction

- Current software projects are very complex
- They can involve hundreds of people and tasks
- An efficient way of assigning employees to tasks is required
- An automatic software tool can assist to the software project manager
- **Problem**: assign employees to tasks with a given dedication degree

**Employee**
- Salary
- Maximum dedication
- Skills

**Task**
- Effort
- Required skills
- TPG

- What is the performance of metaheuristics when the problem size increases?
Problem Formulation: duration

- Project duration (computation)

<table>
<thead>
<tr>
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\[ \sum E = 0.8 \]

Effort T2 = Duration T2

Gantt diagram of the project
### Problem Formulation: cost

- Project cost (computation)

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**Time employee E3 spends on task T4**
### Problem Formulation: cost

- **Project cost (computation)**

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\[ \sum = \text{time the employee spends on the project} \]

- **Salary of E3**

- **Cost of employee E3 due to its participacion**
## Problem Formulation: cost

- **Project cost (computation)**

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\[
\sum = \text{Project cost}
\]

- Cost of employee E1 due to its participation
- Cost of employee E2 due to its participation
- Cost of employee E3 due to its participacion
- Cost of employee E4 due to its participacion
Problem Formulation: constraints

- **Constraints**

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\[ \sum \text{ skills of the task they perform} > 0 \]

**R1.** All tasks must be performed

**R2.** The union of the work team skills must include the required skills of the task they perform
Problem Formulation: constraints

- Constraints (cont.)

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R3. No employee must exceed her/his maximum dedication
Algorithms in the comparison

- **NSGA-II**
  - Generational GA
  - Ranking & Crowding

- **PAES**
  - (1+1) Evolution Strategy + External Archive
  - Adaptive Grid

- **DEPT**
  - Differential Evolution
  - Pareto Tournament

- **MO-FA**
  - Firefly Algorithm
  - Light intensity & Firefly attraction
Algorithms: NSGA-II

1: proc Input:(nsga-II)  //Algorithm parameters in ‘nsga-II’
2: P ← Initialize_Population()  // P = population
3: Q ← ∅  // Q = auxiliary population
4: while not Termination_Condition() do
5:     for i ← 1 to (nsga-II.popSize / 2) do
6:         parents←Selection(P)
7:         offspring←Recombination(nsga-II.Pc,parents)
8:         offspring←Mutation(nsga-II.Pm,offspring)
9:     Evaluate_Fitness(offspring)
10:     Insert(offspring,Q)
11: end for
12: R ← P ∪ Q
13: Ranking_And_Crowding(nsga-II, R)
14: P ← Select_Best_Individuals(nsga-II, R)
15: end while
16: end_proc
Algorithms: PAES

1: proc Input:(paes) //Algorithm parameters in ‘paes’
2: archive ← ∅
3: currentSolution ← Create_Solution(paes) // Creates an initial solution
4: while not Termination_Condition() do
5:     mutatedSolution ← Mutation(currentSolution)
6:     Evaluate_Fitness(mutatedSolution)
7:     if IsDominated(currentSolution, mutatedSolution) then
8:         currentSolution ← mutatedSolution
9:     else
10:        if Solutions_Are_Nondominated(currentSolution, mutatedSolution) then
11:            Insert(archive, mutatedSolution)
12:            currentSolution ← Select(paes, archive)
13:        end if
14:     end if
15: end while
16: end_proc

5.2. Test Data Generator

Our test data generator breaks down the global objective (to cover all the branches) into several partial objectives consisting of dealing with only one branch of the program. Then, each partial
Algorithms: DEPT

1: proc Input:(dept) //Algorithm parameters in ‘dept’
2: P ← Initialize_Population()
3: while not Termination_Condition() do
4:    for i ← 1 to dept.popSize do
5:       Randomly select three different indices $i_1$, $i_2$ and $i_3$
6:       $v = P[i_1] + \lambda \cdot (Best - P[i]) + F \cdot (P[i_2] - P[i_3])$ // Trial vector
7:       $u$ = Recombine($v$, $P[i]$)
8:       Evaluate($u$)
9:       $P[i] = $ Replacement($u$, $P[i]$) // Taking into account Pareto dominance
10:   end for
11: end while
12: end_proc

$$MOF(i) = IsDominated(i) \times PS + Dominates(i)$$
Algorithms: MO-FA

Objective function $f(x)$, $x = (x_1, ..., x_d)^T$

Generate initial population of fireflies $x_i$ ($i = 1, 2, ..., n$)

Light intensity $I_i$ at $x_i$ is determined by $f(x_i)$

Define light absorption coefficient $\gamma$

while ($t < \text{MaxGeneration}$)
  for $i = 1 : n$ all $n$ fireflies
    for $j = 1 : i$ all $n$ fireflies
      if ($I_j > I_i$), Move firefly $i$ towards $j$ in $d$-dimension; end if
      Attractiveness varies with distance $r$ via $\exp[-\gamma r]$
      Evaluate new solutions and update light intensity
    end for $j$
  end for $i$
end while

Rank the fireflies and find the current best

Postprocess results and visualization

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha (\text{rand} - \frac{1}{2})$$
Algorithms: repair mechanism

- For constraint R3

\[ x'_{ij} = \frac{x_{ij}}{\max_i,\tau \{e_{i,\text{work}}(\tau)\} + \varepsilon} \]

\begin{align*}
\text{Overwork} & \quad \text{Maximum dedication} \\
\text{Dedication} & \quad \text{Time} \\
\text{Repair mechanism} & \\
\text{Maximum dedication} & \\
\text{Dedication} & \quad \text{Time}
\end{align*}
Experiments: Quality Indicators

- **Hypervolume (HV)**
  - *Volume covered* by members of the non-dominated set of solutions
  - Measures both *convergence and diversity* in the Pareto front
  - Larger values are better

- **Attainment surfaces**
  - *Localization statistics* for fronts
  - The same as the *median* and the *interquartile range* in the mono-objective case
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Experiments: Instances and Parameters

Problem instances

- 36 instances of increasing number of tasks (16-512) and employees (8-256)
- Labeled as i<tasks>-<employees>

Global Parameters

- Stopping condition: 100 000 function evaluations
- Approximated Pareto front size: 100 solutions
- 100 independent runs for each algorithm-instance
- Statistical tests for significance differences
- Representation: vector of real numbers
Experiments: Algorithm-Specific Parameters

**NSGAII**
- Population: 100
- SBX ($\eta_c=20$, $p_c=0.9$)
- Polynomial mutation ($\eta_m=20$, $p_m=1/L$)

**PAES**
- Population: 1
- Polynomial mutation ($\eta_m=20$)

**DEPT**
- Population: 32
- RandToBest/1/ Binomial
- CR=0.9, F=0.5

**MO-FA**
- Population: 32
- Mutation factor: 0.5
Introduction

Software Project Scheduling

Algorithms

Experiments & Results

Conclusions & Future Work

Results: Hypervolume Comparison

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<tr>
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<th>PAES</th>
<th>DEPT</th>
<th>MO-FA</th>
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</thead>
<tbody>
<tr>
<td>i16-8</td>
<td>0.661±0.028</td>
<td>0.732±0.019</td>
<td>0.311±0.020</td>
<td>0.540±0.022</td>
</tr>
<tr>
<td>i16-16</td>
<td>0.468±0.026</td>
<td>0.826±0.013</td>
<td>0.327±0.038</td>
<td>0.608±0.031</td>
</tr>
<tr>
<td>i16-32</td>
<td>0.147±0.016</td>
<td>0.809±0.009</td>
<td>0.220±0.037</td>
<td>0.379±0.088</td>
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<tr>
<td>i16-64</td>
<td>0.129±0.017</td>
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<td>0.287±0.034</td>
<td>0.370±0.092</td>
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<tr>
<td>i16-128</td>
<td>0.048±0.013</td>
<td>0.722±0.012</td>
<td>0.122±0.028</td>
<td>0.139±0.075</td>
</tr>
<tr>
<td>i16-256</td>
<td>0.018±0.008</td>
<td>0.682±0.010</td>
<td>0.078±0.029</td>
<td>0.069±0.029</td>
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<tr>
<td>i32-8</td>
<td>0.538±0.029</td>
<td>0.721±0.017</td>
<td>0.107±0.024</td>
<td>0.209±0.029</td>
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<tr>
<td>i32-16</td>
<td>0.190±0.026</td>
<td>0.820±0.012</td>
<td>0.035±0.017</td>
<td>0.341±0.045</td>
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<td>i32-32</td>
<td>0.121±0.014</td>
<td>0.743±0.009</td>
<td>0.109±0.018</td>
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<td>i32-64</td>
<td>0.049±0.010</td>
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<td>i32-128</td>
<td>0.041±0.007</td>
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<td>0.080±0.015</td>
<td>0.061±0.011</td>
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<tr>
<td>i32-256</td>
<td>0.009±0.007</td>
<td>0.617±0.016</td>
<td>0.011±0.011</td>
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<tr>
<td>i64-8</td>
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<td>0.089±0.014</td>
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<td>0.221±0.022</td>
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<td>i64-32</td>
<td>0.063±0.011</td>
<td>0.798±0.008</td>
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<td>0.073±0.012</td>
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<td>i64-256</td>
<td>0.000±0.000</td>
<td>0.618±0.013</td>
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<td>i128-8</td>
<td>0.320±0.019</td>
<td>0.986±0.006</td>
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<td>0.076±0.031</td>
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<td>i128-16</td>
<td>0.253±0.023</td>
<td>0.988±0.013</td>
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<td>0.980±0.010</td>
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<td>i128-64</td>
<td>0.095±0.008</td>
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<tr>
<td>i128-128</td>
<td>0.029±0.008</td>
<td>0.782±0.009</td>
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<td>0.000±0.000</td>
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<tr>
<td>i256-8</td>
<td>0.309±0.015</td>
<td>0.998±0.002</td>
<td>0.000±0.000</td>
<td>0.010±0.019</td>
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<tr>
<td>i256-16</td>
<td>0.156±0.012</td>
<td>0.987±0.007</td>
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<td>i256-64</td>
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<td>i256-128</td>
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<td>i256-256</td>
<td>0.003±0.003</td>
<td>0.626±0.016</td>
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<td>i512-8</td>
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<tr>
<td>i512-16</td>
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<td>0.102±0.003</td>
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<td>0.560±0.031</td>
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Hypervolume (HV)

- PAES is the clear winner in HV
- MO-FA is the second best for small instances
- NSGA-II is the second best for large instances
- DEPT is the worst algorithm in the comparison
Results: 50%-Empirical Attainment Surfaces (EAS)
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Results: 50%-Empirical Attainment Surfaces (EAS)
Conclusions & Future Work

Conclusions

- PAES is the algorithm with **better scalability behaviour**
- MO-FA is the **second best in small instances**
- NSGA-II is the **second best in large instances**
- DEPT is the **worst algorithm** in the comparison
- HV **not always provides enough information** to determine the best algorithm. **EAS** is a good alternative.

Future Work

- Use **real-world instances** of the problem
- Change **the formulation of the problem** to get closer to reality
On the Scalability of Multi-objective Metaheuristics for the Software Scheduling Problem

Thanks for your attention !!!