A Comparative Study of the Effect of Parameter Scalability in Multi-Objective Metaheuristics

IEEE Congress on Evolutionary 2008, June 1-6, 2008

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May 27, 2008
Outline of the talk

1. Introduction
   - Motivation
   - Contributions

2. Experimentation
   - Parameterization and Methodology
   - Scalability and speed
   - Discussion

3. Conclusions and Future Work
## Motivation

### Parameter scalability in MO metaheuristics
- MO community is paying much attention to **many-objective optimization**
- Few works devoted to **dimensionality** (number of decision variables)
- Real-world engineering problems → thousands of variables
- Performance assessment of MO algorithms
  - Standard benchmarks: usually, 30 decision variables
  - Only 10 decision variables for the hardest problems (e.g., ZDT4)

### Convergence speed of MO metaheuristics
- A predefined maximum number of function evaluations is normally used
- **Goal**: study computational effort to reach the true Pareto front
- Practical interest when solving **time-consuming MOPs**
Contributions of this work

- Scalability and convergence speed in-depth analysis of MO metaheuristics
- Six state-of-the-art algorithms
  - Three GAs: NSGA-II, SPEA2, and PESA-II
  - An evolution strategy: PAES
  - A particle swarm optimization: OMOPSO
  - A cellular GA: MOCell
- The ZDT problem family
  - Keep the same Pareto front with different number of decision variables
  - Experiments that range from 8 up to 2048 decision variables
Parameterization and methodology

**Parameterization**

- Aimed at **fairly comparing** the algorithms (e.g. using the same population size)
- They all are implemented using **jMetal** (http://neo.lcc.uma.es/metal)

**Methodology**

- The **stopping condition** is either
  - 500,000 function evaluation, or
  - Convergence criterium reached → the **Hypervolume** indicator
- The **convergence criterium** in-depth
  - Reach **95%** of the HV value of the **true Pareto front** of the problem
  - Every **100** function evaluations
    - Nondominated solutions at each generation of NSGA-II and SPEA2
    - The external archive of PESA-II, PAES, and MOCell
    - The leaders archive of OMOPSO
Analysis of the ZDT1 problem

Scalability
- Up to 512 variables: all the algorithms reached 95% HV of true Pareto front
- 1024 variables: MOCell and PESA-II failed
- 2048 variables: only PAES

Speed
1. Up to 256 variables: OMOPSO
2. More than 512 variables: PAES
Analysis of the ZDT2 problem

Scalability

1. Up to 512 variables: the same as ZDT1
2. 1024 variables: just OMOPSO and PAES successfully solved this MOP
3. 2048 variables: again, only PAES

Speed

1. Up to 512 variables: OMOPSO
2. More than 512 variables: PAES
Analysis of the ZDT3 problem

**Scalability**
- Up to 512 variables: all the algorithms
- 1024 variables: MOCell, PESA-II, and OMOPSO failed
- 2048 variables: again, only PAES

**Speed**
1. Up to 64 variables: OMOPSO
2. 128 to 1024 variables: NSGA-II
Analysis of the ZDT4 problem

Scalability
- OMOPSO has never converged
- MOCell reached the best results up to 128 variables
- More than 128 variables: no solver converged

Speed
- MOCell is the fastest
- PESA-II is the second best
Analysis of the ZDT6 problem

Scalability

- Up to 128 variables: all the algorithms converged
- OMOPSO and PAES successfully solved the MOP up to 1024 variables
- 2048 variables: no algorithm converged

Speed

- OMOPSO is the fastest
- PAES is the second best
Global results

### Scalability: address the problems with the higher number of decision variables

<table>
<thead>
<tr>
<th>Ranking</th>
<th>ZDT1</th>
<th>ZDT2</th>
<th>ZDT3</th>
<th>ZDT4</th>
<th>ZDT6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PAES</td>
<td>PAES</td>
<td>NSGA-II</td>
<td>MOCell</td>
<td>OMOPSO</td>
</tr>
<tr>
<td>2</td>
<td>OMOPSO</td>
<td>OMOPSO</td>
<td>SPEA2</td>
<td>PESA-II</td>
<td>PAES</td>
</tr>
<tr>
<td>3</td>
<td>NSGA-II</td>
<td>MOCell</td>
<td>PAES</td>
<td>NSGA-II</td>
<td>MOCell</td>
</tr>
</tbody>
</table>

### Speed: lowest number of function evaluation to reach 95% HV of the true PF

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<th>ZDT3</th>
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<th>ZDT6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OMOPSO</td>
<td>OMOPSO</td>
<td>NSGA-II</td>
<td>MOCell</td>
<td>OMOPSO</td>
</tr>
<tr>
<td>2</td>
<td>PAES</td>
<td>PAES</td>
<td>OMOPSO</td>
<td>PESA-II</td>
<td>PAES</td>
</tr>
<tr>
<td>3</td>
<td>NSGA-II</td>
<td>MOCell</td>
<td>OMOPSO</td>
<td>NSGA-II</td>
<td>MOCell</td>
</tr>
</tbody>
</table>

When an algorithm scales well, it usually requires a low number of function evaluation to converge towards high quality Pareto fronts.
Conclusions and future work

Conclusions

- Study of 6 MO metaheuristics over the parameter-wise, scalable ZDT family
  - Scalability analysis
  - Convergence speed

- Results
  - PAES scales the best and it converges the second best
  - OMOPSO converges the fastest, but it fails with multifrontal MOPs
  - MOCell fits specially well on ZDT4 (the most difficult MOP)

Future Work

- Include a wide study with more scalable MOPs
- Design new MO metaheuristics to overcome the limitations of those in current use
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Questions?