MODELS FOR PICK-UP AND DELIVERIES FROM DEPOTS WITH LASSO SOLUTIONS

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ABSTRACT

Vehicle Routing Problems (VRP) are encountered in many practical situations. VRP with the added complexity of pick-up ***and*** *deliveries will make the planning problem more difficult. In this paper, we will consider a restricted situation where all delivery demands start from the depot and all pick-up demands shall be brought back to the depot. In a traditional VRP setting this can lead to bad utilisation of the vehicles capacities, increased travel distances or a need for more vehicles. An alternative is to relax the VRP restriction that all customers shall only be visited once. This relaxation can lead to several different routing options. One such solution can be a so-called lasso-solution. In such a solution the first customers on the route are visited twice. On the first visit, only the delivery demands are performed, hence creating more free space on the vehicle. On the way back to the depot, the same customers are visited the second time, now in order to perform the pick-up service. Not very much research has been done on this kind of situation and a lot of new problems and possibilities arise. In this paper we discuss some of these problems and offer different heuristics for finding solutions for the problem. The heuristics will be based on variations of well-known heuristics for the traditional VRP.*

Key Words: Vehicle Routing Problem, Pick-up and Delivery, Lasso-solutions, Heuristics

1. INTRODUCTION

The Vehicle Routing Problem (VRP) and its specialisation, the Travelling Salesman Problem (TSP), are well-studied combinatorial optimisation problems with many practical applications. Both VRP and TSP are NP-complete problems and as such they will in general be difficult solve to optimality when the problems become large. For overviews of the said problems, see f. ex. Bodin & al. (ed.), (1983), Lawler & al. (ed.), (1985), Golden and Assad (ed.), (1988) and more recently Ball & al. (ed.), (1995), chapter 4 in volume 7 or Ball & al. (ed.), (1995), chapters 1 - 4, volume 8.

Basically the VRP describes a planning situation where a fleet of vehicles serves a set of customers, starting from a depot, visiting each customer one and only one time and then return to the depot. The customers' demands are supposed to be known and the capacities of the vehicles taken together must be equal to or exceed the customers' demands. Many extensions of this basic planning situation have been done through the last decades. One such extension can be that the customers' demands are a mix of pick-up and delivery demands. A mathematical model for this general situation can be found in Golden and Assad, (1988), in the article by Desrochers & al., pp. 65 - 84.

The present paper concerns a planning situation where pick-ups and deliveries are important parts of the problem, but restricted to situations where all the demands - that is both the pick-up and the delivery demands - end and start, respectively, at the depot. Hence, there are no interchanges of goods between the customers.

Let i = 2,3,......., n denote the customers and i = 1 denote the depot. Further, let d_i and p_i, $i = 2,3,......,n$ denote the delivery and the pick-up demands of the customers, respectively. Then three different situations can occur:

Situation 1: For every customer i, $d_i \geq p_i$.

In this case any feasible solution to the planning problem using the delivery demands as in-put parameters and disregarding the pick-up demands, will be a feasible solution to the extended problem of taking the pick-up demands into consideration as well.

Situation 2: For every customer i, $d_i \leq p_i$.

In this case any feasible solution to the planning problem using the pick-up demands as in-put parameters and disregarding the delivery demands, will be a feasible solution to the extended problem.

Situation 3: For some customers $d_i \leq p_i$, for other customers it is the other way around.

In this case we cannot use only one set of parameters, since a feasible solution for one set of parameters only, can very well be infeasible when the other set of parameters is taken into consideration. Hence, the problem must be solved taking both sets of parameters into consideration at the same time. This can be done for example by using the model suggested by Desrochers & al., mentioned above, with a few specialisations in the model.

However, the classical VRP and TSP with or without the problem of delivery and pick-up demands, have one common tacit premise: *Every customer is visited one and only one time during the time horizon.* In the case of a pick-up and delivery problem, this premise would lead to an optimal solution more expensive than in a pure delivery or pure pick-up situation, since the sequence of visiting the customers for one or more of the involved vehicles probably will change.

The main objective of this paper is to relax the premise that every customer is visited only once, or more precisely, we accept one or two times. Hence, the following two situations can occur:

A customer visited twice, will either be visited by two different vehicles or be visited twice by the same vehicle.

The former case is usually called *split-delivery*. A description of such a planning situation can be found in Brenninger-Göthe, (1989). However, the premise of split-deliveries is restricted to considering delivery demands (or pick-up demands) only and the multiple visits - as mentioned above – are performed by *different* vehicles for the customers involved.

This paper will consider a planning situation with *delivery and pick-up* demands from the depot, where some of the customers can be visited *twice* by the *same* vehicle.

This type of solution to a difficult operational planning situation is used in practise for instance in the beverage industry in Norway and probably in other industries and countries as well. For reviews of the problem see Halskau and Løkketangen, (1997), Haukebø et al., (1998) and Tjøstheim, (1999). Basically, a vehicle visits the first customer (or a few more) twice. At the first visit only the delivery service is performed. Then all the other customers are visited in some order and both types of services are performed. Then the vehicle returns to perform the pick-up deliveries for the first customers.

This way of performing the vehicle routing creates - not sub-cycles as in a traditional VRP solution - but lassos. In the description above the lasso will consist of a rather large noose (or *loop*) and a short *spoke* with the depot at its end. In principle the loop can of course be made smaller and the spoke longer, without violating the underlying principle. The necessary knot used for making the loop is, in the lasso context, denoted as a 'honda'. However, in our context the term '*junction*' seems to be more appropriate and will be used below. The nodes belonging to the spoke are denoted *spoke nodes* and the nodes belonging to the loop, *loop nodes*.

The rest of this paper is organised as follows: In section 2 a more detailed discussion of different aspects of the lasso solution is discussed. In section 3 we discuss different aspects of what a feasible solution is in the pick-up and delivery setting. In section 4 we offer some heuristics for this new situation, based on the lasso solution and the feasibility concepts in the section 3. Section 5 gives examples for the proposed heuristics and in section 6 some thoughts about further research are offered.

2. THE LASSO SOLUTION

Initially we will restrict ourselves to a situation where there is only one vehicle involved, that is, a situation directly comparable to the TSP. We will assume that the vehicle's capacity is sufficiently large to handle both delivery demands and the pick-up demands separately. Further, we will assume that we have a situation as described in situation 3 in section 1, that is, for at least some customers one type of demand can exceed that of the other type and for other customers it can be the other way around. Further, we will assume a complete undirected graph $G = (E, N)$ where E denotes the edge set and N the set of nodes.

If the vehicle leaves the depot fully loaded and the route is done in a traditional TSP way, the driver may encounter certain problems. Firstly, he may sooner or later find himself in a situation where the number of units to be loaded onto the truck is larger than the truck can handle. Secondly, he may have to unload some of the goods to get access the units he wants to deliver to a given customer. This takes time. In order to circumvent such situations the dispatcher or the driver can try to find another sequence of customers – hoping against hope – that the first customers in this new sequence have less pick-up demands than delivery demands. In this way sufficient free space on the truck can be created in order to serve the customers to come. This will usually cost some money in terms of larger driving distances and more time spent. Alternatively, the truck can leave the depot not fully loaded in order to have enough free space to handle the problem of the more or less stochastic pick-up demands. In this way the driving distance can be kept at a minimum, but the vehicle will not be fully utilised and can lead to a need for another vehicle. So, in any case, using a traditional TSP cycle may not be practical or the Hamiltonian cycle used in the TSP solution is not feasible and must be changed, incurring some extra cost.

Using a so-called lasso can solve some of the problems encountered. If we accept that some of the customers can be visited twice and these customers are the first to be visited, then sufficient space can be created on the vehicle by only performing the delivery service at the first visit, and making the pick-ups at the second visit. The cost of travelling along a lasso can of course be larger than the optimal Hamiltonian cycle, but may be smaller than the cheapest feasible Hamiltonian cycle. Furthermore, it may simplify the problem of loading and unloading the vehicle, since the access to the delivery demands will be easier and the stability of the different routes may increase. In figure 1 below a lasso for one vehicle is shown. The lasso has one spoke node, 5 loop nodes and of course one junction only. The depot is depicted as a square.

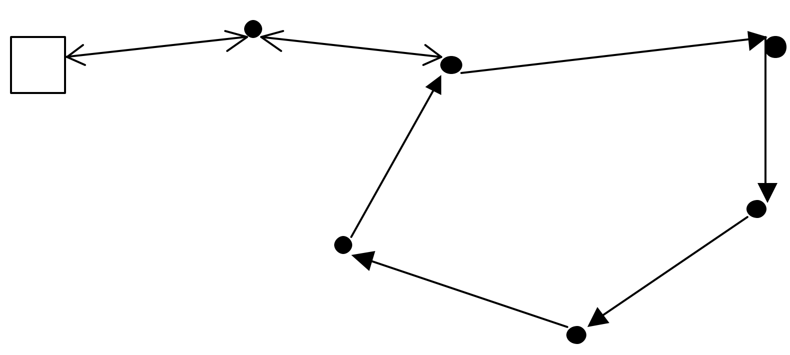

Figure 1. An example of a lasso solution

Note that a Hamiltonian cycle can be regarded as a special case of a lasso, that is a lasso with no spoke or junction. Another extreme case of a lasso will be a lasso with no loop nodes, i.e. a path. We need exactly n edges for every feasible lasso solution. Note, that the lasso is one-tree, graph theoretically speaking. Hence, the *minimal one-tree* will be a lower bound for the optimal solution, but this will probably be a poor lower bound because in the practical application the edges on the spoke has to be used twice. If the triangle inequality holds for the underlying network, it is obvious that the TSP solution – disregarding the delivery and pick-up parts of the problem – is a lower bound for the optimal lasso solution. In Halskau, (2000) one can find an exact IP-model for finding an optimal lasso solution for one vehicle.

3. FEASIBILITY OF VRP SOLUTION WITH PICK-UP AND DELIVERY DEMANDS

As mentioned above, in classical VRP, a set of customers with known demands is to be serviced by a fleet of vehicles from a depot and the objective is to find a set of routes - a route configuration - that service the customers such that the vehicles' capacities are not violated and the total distance covered by the entire fleet is minimised. It is tacitly assumed that the routes form sub-cycles all containing the depot. Hence, each customer is visited by exactly one vehicle. Thus, a feasible solution to the VRP – a feasible route configuration - consists of a set of feasible routes, one for each vehicle.

In a pure delivery situation feasibility of a route depends only on the total quantity of goods, delivered to the customers along the route. Suppose, a vehicle starts from the depot (i = 1) and travel along a certain path $1-i_1-i_2-...-i_j-...$ until it reaches node i_k. With the *cumulative delivery* $C_d(i_k)$ *at the point* i_k of the path we understand the total quantity of goods, delivered to all customers along the path up to and including node i_k, that is

$$(3.1) \qquad C_d(i_k) = \sum_{i \in P(1,i_k)} d_i \,,$$

where $P(1, i_k)$ denotes the nodes along the path. The path becomes infeasible as soon as the vehicle can not perform the delivery service at a next customer i_{k+1} because the total cumulative delivery will exceed the vehicle's capacity K_v, i.e. as soon as

$$(3.2) \qquad C_d(i_k) \le K_v \text{ and } C_d(i_{k+1}) > K_v \,.$$

A feasible path creates a feasible route by connecting the last node i_k with the depot.

The cumulative delivery function value increases monotonously along the route starting from zero at the depot and obtaining its maximal value at the last visited customer i_k on the route. We call this value *maximal cumulative delivery* of the route. The route feasibility for the pure delivery VRP means that the maximal cumulative delivery of the route does not exceed the vehicle's capacity. We will refer to this type of route feasibility as "*delivery-feasibility*".

Let $L(i_k)$ be the *vehicle's load* just after leaving node i_k. Assume that the vehicle leaves the depot with an initial load $L(1)$ less or equal to the vehicle's capacity. Then the vehicle's load at the node i_k of the route is a difference between the initial load and the cumulative delivery at this node, i. e. $L(i_k) = L(1) - C_d(i_k)$. The value of the initial load in practice is equal to the maximal cumulative delivery of the route. The vehicle's load function in the pure delivery VRP monotonously decreases along the route from the initial load value at the depot to the minimal zero value at the last customer of the route.

In a similar way as for the delivery demand, we now define the *cumulative pick-up* $C_p(i_k)$ *at the point* i_k. By this we understand the total quantity of goods, picked-up from all customers along the path up to and including node i_k, that is

$$(3.3) \qquad C_p(i_k) = \sum_{i \in P(1, i_k)} p_i .$$

The path becomes infeasible as soon as the vehicle can not perform the pick-up service at a next customer on the path i.e.

$$(3.4) \qquad C_p(i_k) \le K_v \text{ and } C_p(i_{k+1}) > K_v .$$

The feasible path creates the feasible route by connecting the last node i_k with the depot. The vehicle's load at every node i_k in this case has the same value as the cumulative pick-up, i. e. $L(i_k) = C_p(i_k)$. This function monotonously increases along the route from the zero at the depot to the maximal value at the last customer of the route. We call it *maximal cumulative pick-up* of the route. The route feasibility for a pure pick-up VRP means in fact that the maximal cumulative pick-up of the route does not exceed the vehicle's capacity. We will refer to this type of route feasibility as "*pick-up-feasibility*".

In the single-vehicle version of the pure VRP, the assumption that the total delivery (or pick-up) demand does not exceed the vehicle's capacity, i.e.

$$(3.5) \qquad \sum_{i \in N} d_i \le K_v \text{ or } \sum_{i \in N} p_i \le K_v$$

guarantees the route feasibility for any Hamiltonian-cycle. The delivery or pick-up -feasibility condition (3.5) are *necessary and sufficient* conditions for route feasibility in a pure VRP setting. We can see that the delivery or pick-up feasibility of the route depends only on the set of customers assigned to the vehicle. It will not depend on the sequence the customers are visited in. In the VRP with delivery *and* pick-up demands the situation is more complex. Here, the vehicle's capacity can be violated at any node of the route. Such a violation will depend on the sequence of the customers. The delivery- *and* pick-up-feasibility conditions are *necessary* conditions for route feasibility, but not *sufficient* conditions.

The vehicle load at any point of the route in this mixed-demand problem is a function of the cumulative delivery, the cumulative pick-up, and the initial load value, namely

$$(3.6) \qquad L(i_k) = C_p(i_k) + L(1) - C_d(i_k) .$$

Therefore, even when each of cumulative demands (3.1) and (3.3) at any node i_k of the path do not exceed the vehicle's capacity, the vehicle's load given by (3.6) can exceed the vehicle's capacity. This means that the path becomes infeasible because the vehicle can not perform service at a next customer i_{k+1} on the path i. e.

$$(3.7) \qquad L(i_k) \le K_v \text{ and } L(i_{k+1}) > K_v \,.$$

As we see, in the mixed-demand VRP situation there is a new type of route feasibility that must be obeyed. This feasibility will depend on the sequencing of the customers. We will refer to this type of route feasibility as "*load-feasibility*". In the mixed-demand VRP situation all the three types of feasibility must be obeyed. That is, a route is feasible if and only if it is delivery-feasible, pick-up-feasible, and load-feasible.

There have been several attempts to develop heuristics for the pick-up and delivery VRP. These are usually modifications of well-known procedures for the basic VRP such as the saving heuristics, insertion procedures, space filling curves, tour-partitioning procedures. For an overview of such heuristics for pure VRP heuristics, see for example several articles in Golden and Assad (1988). In all these attempts to modify the classical heuristics some severe simplifications are done. One such simplification is that a given node is either a delivery node or a pick-up node, see Mosheiov (1998). This will hardly be the case in practical applications. Other simplifications are done by assuming that the pick-up demands are substantially less than the delivery demands or that the nodes with pick-up demands are postponed to the end of the route. These heuristics solve the load-feasibility problem for each vehicle with the help of different restrictions and assumptions, but under the traditional premise that every customer is visited once and only once. Hence, they all end up with sub-cycles. The costs can be significantly larger than those necessary in a pure VRP situation.

However, it is not necessary to take all these assumptions. More precisely, let a set of customer nodes be given such that the cumulative delivery and pick up demands are less than a vehicles capacity, that is we have feasibility in the traditional sense. Now, it is always possible to find a sub-cycle for the vehicle that is delivery-, pick-up- and load-feasible. This can be achieved in the following way.

First, split the set of nodes into two subsets. The first set should consist of all nodes where the delivery demands are strictly larger than the pick-up demands. Let the second set consists of the remaining nodes, i.e. nodes where the pick-up demands are larger or equal to the delivery demands. Now, let the vehicle leave the depot with all the delivery demands and visit all the nodes in the first set in any order performing both kind of services. When this is done, the vehicle travels to any node in the second set and visit the remaining nodes in any order, performing both services and then return to the depot. As long as a vehicle perform the services in the first set, the load function $L(i)$ will be monotonously decreasing. Switching to the second set, the load function will be monotonously increasing and ending up with the sum all the pick-up demands as its end value. Hence the vehicle's capacity will never be violated. This means that a classical solution to the mixed problem is always possible as soon the both of the two traditional criteria in (3.5) are fulfilled that is (3.5) with 'and', but the costs can increase.

Allowing customer's service to be performed by the same vehicle twice at the different points of the route, it can easily happen that the shortest feasible lasso-route becomes cheaper than the shortest feasible Hamiltonian cycle for the same set of customers. For an example, see Halskau, (2000).

A lasso-route has to conform to feasibility in the same way as a cycle-route, but the vehicle's load along the lasso changes in another way. Along the spoke nodes out from the depot, up to and including the junction node, the vehicle's load function decreases monotonously from the initial load value, $L(1)$. In the opposite direction it increases, reaching the maximal cumulative pick-up when leaving the last node before returning to the depot. At the loop nodes the vehicle's load value can change unpredictably, as in the classical situation, but will start at a lower value. This gives a larger probability for a lasso route to become feasible. The difference between the vehicle's load average value and the vehicle's capacity value at the loop nodes will increase as the spoke length increases. In the extreme case, we may use a lasso with no loop nodes at all, i.e. a path. Along a path where every customer are visited twice, apart from the last one, the vehicle's load function will monotonously decrease on the way out from the depot and increase on the way back. Thus, it is always possible to create the feasible lasso-route by varying the length of the spoke. Lassos allow greater flexibility in creating feasible routes. Hence, the shortest feasible routes for some vehicles may be sub-cycles, for others – lassos.

4. HEURISTIC METHODS FOR CONSTRUCTING LASSOS

In this section different heuristics for finding lasso solutions are proposed. These heuristics are based on well-known heuristics for the traditional VRP. The proposed heuristics can come in many different variations. We have restricted ourselves to some basic ideas.

4.1. Heuristics for a single vehicle

All known tour construction heuristics for the pure VRP/TSP create the delivery- (pick-up) feasible routes in the form of a Hamiltonian cycle. Some of the tour construction heuristics, such as the nearest neighbour or the sweep-algorithm create the tour starting from the depot - node by node. They start with a single node and sequentially create a path according to some criterion until a complete tour is obtained. Insertion heuristics construct a tour starting with a sub-cycle. A sub-cycle is extended by adding new nodes, one at a time. The chosen node is inserted into the existing sub-cycle by some criterion and hence creating a new and a larger sub-cycle until it includes all nodes. One of the most well known among iterative construction methods and most widely used in practice, is probably the saving heuristic. Here a path is created by choosing the arc according to the so-called saving value until all nodes are connected and then the path is closed by connecting the end nodes to the depot. The majority of these procedures give no possibility to check the vehicle's load at every point of the tour until it's construction is finished. For the pure VRP there is no need to do it - a priori at any point of the Hamiltonian tour the vehicle's load doesn't exceed the vehicle's capacity. For the mixed-demand VRP the situation is quite different. It is impossible to check route feasibility - specifically load-feasibility - until the whole tour is finished. Thus, it is not possible to use all these classical heuristics directly when constructing feasible routes in a mixed situation.

The exceptions are the tour construction procedures, creating the tour node by node starting from the depot, hence specifying a sequence. To specify a sequence is necessary if one wants to check load feasibility. We propose a heuristic based on the adaptation of the nearest neighbour heuristic's ideas and baptise it the *feasible lasso construction procedure*. Initially all the customer nodes are assumed to be loop nodes. The vehicle's initial load is equal to the total customers' delivery demands. The heuristic sequentially creates a path until the vehicle's load exceeds the vehicle's capacity and changes the status of one node and its service so that the created path becomes load-feasible. When all customer nodes are in the path and the status of each node on the path is identified, nodes are connected in the required sequence so creating a lasso.

The feasible lasso construction procedure:

Step 1: Use the nearest neighbour heuristic to find a path such that i_k and i_{k+1} satisfy (3.7) and $L(i_j) \le K_v; \forall j < k$. If no such nodes i_k and i_{k+1} are found, return to the depot from the last node on the path and stop. If such nodes i_k and i_{k+1} are found, change the first loop node i_1 into a spoke node, ignore its pick-up order and reduce the vehicle's load value at the path nodes by the pick-up demand p_{i_1}. Go to step 2.

Step 2: Use the nearest neighbour heuristic to extend the path until a new pair i_k and i_{k+1} is found satisfying (3.7) and $L(i_j) \le K_v; \forall j < k$. Change the first loop node on the path into a spoke node and ignore its pick-up order. Reduce the vehicle's load value at the path nodes by the pick-up demand of the new spoke node.

Step 3: Repeat Step 2 until the path includes all the nodes. Connect the end node of the path with the last created spoke node. Change this node into a junction node. Connect nodes according to their status and create the lasso.

The proposed heuristic can come in many different variations. Other criteria for the choice of the next node on the path can be used instead of the 'nearest neighbour' criterion.

The lasso construction procedure can be easily adapted for a situation when an initial route is given. Assume that the VRP has been solved either to optimality or by a heuristic disregarding customers' demands. Assume that this solution is not load-feasible. We now propose a procedure for "*re-constructing*" this initial tour into a feasible lasso solution. In the lasso solution, the vehicle will perform *delivery service* to the customers in the same sequence as on the basic tour. However, the *pick-up service* will not be performed simultaneously for all the customers. The vehicle will visit these customers a second time and perform pick-up service in the reverse sequence after all the deliveries are made. Initially all the customer nodes are assumed to be loop nodes. The vehicle's initial load is equal to the total of the customers' delivery demands. This heuristic is referred to as:

The lasso improvement procedure:

Step 0: Make a Hamiltonian cycle disregarding the customers' demands. Check if this basic tour is load feasible or not. If it is, stop. If not, go to step 1.

Step 1: Start from the depot node, perform the full service along the basic tour until at some node the vehicle's load exceeds the vehicle's capacity. Change the first loop node on the tour after depot into a spoke node, and ignore its pick-up order. Reduce the vehicle's load value at the visited nodes by the pick-up demand of the spoke node. Go to step 2.

Step 2: Continue the service along the basic tour until at some node the vehicle's load exceeds the vehicle's capacity again. Go back, change the first loop node after the spoke on the basic tour into a spoke node and ignore its pick-up order. Reduce the vehicle's load value at the visited nodes by the pick-up demand of the new spoke node. Go to step 3.

Step 3: Repeat Step 2 until all the nodes are visited. Connect the last visited node with the last created spoke node. Change this node into a junction node. Connect all the nodes according to their status and create the lasso. On the way from the junction node to the depot, perform the pick-up service.

4.2. Heuristics for several vehicles

In this sub-section, we consider a situation with more than one vehicle, i.e. a typical VRP situation. Let v denote a vehicle from the vehicle fleet V and K_v the capacity of the vehicle v. We assume that no customer demand exceeds the maximum vehicle capacity. With a feasible route configuration, we will understand a set of feasible routes, one for each vehicle. Let R_v denote the nodes included in the route for vehicle v. We will refer to this feasibility as *"route configuration feasibility"*.

In the pure VRP, the condition that total customers demand (delivery or pick-up) does not exceed the total vehicle capacity, i.e.

$$(4.1) \qquad \sum_{i\in N} d_i \le \sum_{v\in V} K_v \text{ or } \sum_{i\in N} p_i \le \sum_{v\in V} K_v$$

is necessary for a feasible route configuration to exist. Sufficient condition for route configuration existence is that each route R_v is delivery- (pick-up-) feasible, i.e.

$$(4.2) \qquad \sum_{i\in R_v} d_i \le K_v \text{ or } \sum_{i\in R_v} p_i \le K_v \quad \forall v \in V.$$

In the mixed-demand VRP situation (4.1) must be changed from 'or' to 'and'. These are necessary conditions for route configuration feasibility, and the sufficient conditions are that each route is delivery-, pick-up-, and load-feasible.

The process of finding a feasible route configuration consists of two problems, preferably solved simultaneously: 1) The assignment of customers into different clusters (one cluster for one vehicle) and 2) Sequencing the customers in each cluster. Two different main policies are available if one has to split the planning problem into the two parts mentioned above: 'Clustering first, then routing/sequencing' or 'Routing first, then clustering'. In the context of a mixed VRP situation the delivery- and pick-up-feasibility play the main role when clustering takes place. When in routing – the load-feasibility is the main objective.

In many of the known heuristics for pure VRP situations the routing takes place along with the clustering. However, some of theses heuristics, especially the saving and insertion heuristics as well as the improvement and exchange heuristics are difficult to adapt. These heuristics can usually not be used in their original form because they don't guarantee route configuration feasibility or route feasibility (specifically load-feasibility) of the solution. The policy 'routing first, then clustering' works by initially generating a large route, containing all customers, which then is partitioned into a number of smaller routes, all of which are made feasible. This can be quite difficult in a mixed demand situation. The adaptation of heuristics using the 'cluster first, then routing' principle are especially attractive for the mixed-demand VRP, since the routing problem is more complicated in such cases than in the pure VRP due to the load-feasibility problem. In the routing phase, it is possible to use the lasso approach and construct a feasible lasso for each cluster. Heuristics proposed in section 4.1 for single-vehicle problems could be used in the routing phase of the multiple vehicle situation after the clustering has taken place.

Route constructing heuristic for multiple vehicles with pick ups and deliveries:

Step 1: Create a route configuration that satisfies (4.2). Check load-feasibility for each route and vehicle. If all routes are load-feasible, then stop. For each of the routes not load-feasible, go to step 2.

Step 2: Use the lasso improvement procedure on each of the non load-feasible routes from step 1.

The result of this heuristic will to a large degree depend on step 1, but it is not self evident how one can create a route configuration satisfying (4.2). One such possibility is to use the following *sub-routine*:

Step 1: Take a vehicle. Create a path among the nodes not used so far starting from the depot until a pair of nodes i_k and i_{k+1} is found satisfying (3.4). Calculate $C_d(i_k)$. Go to step 2a or 2b.

Step2a : If $C_d(i_k) \leq K_v$, return to the depot. Call this route R_v. $L_v(1) = C_d(i_k)$ Go to step 3.

Step2b : If $C_d(i_k) > K_v$, delete the last nodes on the path one by one until $C_d(i_l) \leq K_v$ for some node i_l. Call this route R_v. $L_v(1) = C_d(i_l)$. Go to step 3.

Step 3 : If any nodes left, go to step 1. If no nodes are left, stop.

For a homogenous vehicle fleet, it is possible to find a lower bound for the number of vehicles required for a solution and thus ensure efficient utilisation of vehicles capacities. If assume that all vehicles are identical with the same capacity K, a simple formula for the minimal number k of vehicles necessary to serve all customers in the mixed-demand VRP can be suggested, namely:

$$(4.3) \qquad k = \max\left\{ \left\lceil \sum_{i \in N} d_i \Big/ K \right\rceil, \left\lceil \sum_{i \in N} p_i \Big/ K \right\rceil \right\} .$$

Given the assumption that vehicle fleet is homogenous and has the size given by (4.3), a heuristic used in step 1 above, can not guarantee route configuration feasibility. Bad utilisation of vehicle's capacity for one cluster can lead to lack of capacity for the remaining clusters. To solve this feasibility problem on the clustering stage several optimisation-based heuristics can be adapted. They are all directly based on one or another mathematical model of the VRP. Among these, we have chosen to focus on the general assignment heuristic of Fisher and Jaikumar (1981) (FJ). The FJ approach is based on a vehicle flow formulation of the basic pure VRP where the number of vehicles is given. Their reformulation of the basic model follows a 'cluster first, then routing' approach. FJ suggested a linear approximation of the cost function in reformulated model. The clustering part of their heuristic is described below and is a relatively simple IP-model, which usually can be solved quite easily.

(4.4) $$\min \sum_{i=2}^{n}\sum_{v=1}^{k} f_{iv}W_{iv}$$

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(4.5) $$\sum_{i=2}^{n} d_i W_{iv} \le K_v \quad \forall v$$

(4.6) $$\sum_{v=1}^{k} W_{iv} = 1 \quad \forall i; i \ne 1$$

The variables W_{iv} specify whether a vehicle v visits a customer i (W_{iv}=1) or not (W_{iv}=0). Coefficients f_{iv} are computed by first choosing a set of "seed-customers", s_v, v = 1,…, k, each one assigned to one vehicle, and then by computing f_{iv} as the cost of inserting customer i into the route of vehicle k from the depot to the seed-customer and back to the depot again, i. e.

(4.7) $$f_{iv} = \min \left\{c_{1i} + c_{is_v} - c_{1s_v}, c_{s_v i} + c_{i1} - c_{s_v 1}\right\}.$$

The coefficient f_{iv} for a seed-customer s_v and the corresponding vehicle v will have the value zero. Seed-customers can be selected either automatically or interactively by the dispatcher. In the case of manual selection, it is suggested that the most distant customers along radial corridors corresponding to major thoroughfares should be chosen. This choice will result in coefficients f_{iv}, which are such that in a solution to the above model customers assigned to the same vehicle are located close to each other.

This solution approach guarantees route configuration feasibility already at the clustering phase. The routing phase is then performed by solving a TSP for each vehicle, v = 1,…, k.

We adapt FJ heuristic for the mixed-demand VRP by reformulating the generalised assignment problem by adding to the model (4.4) – (4.6) the extra constraints

(4.8) $$\sum_{i=2}^{n} p_i W_{iv} \le K_v \quad \forall v$$

Such a model will guarantee not only route configuration feasibility, but also pick-up- and delivery- feasibility of solutions. Never the less, the solution of a TSP for each vehicle over the assigned customers in the routing phase does not guarantee load-feasibility of the routes. We propose to use one of the three possibilities below for construction feasible routes for each vehicle in routing part:

1) Use the exact lasso model from Halskau, 2000 iteratively for construction of feasible lassos.

2) Use the lasso construction heuristic from section 4.1.

3) Find a TSP solution and if necessary reconstruct the sub-cycle into lasso by applying lasso improvement heuristic from section 4.1.

It should be noted that the choice of seed-customers usually is done on spatial considerations only or with a side-glance at customers with very large demands. In the context of load-feasibility problem, customers with large pick-up demands or with large difference between pick-up and delivery demands can also be chosen as seed-customers. How to include the added problem of load feasibility into to the objective is not clear at the present stage.

5. EXAMPLES

In table 5.1 below one will find a cost matrix for a graph with 14 nodes. In table 5.2 there are given pick-up and delivery demands for 13 of the nodes, node 1 being the depot.

Table 5.1. Cost matrix for the graph

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	-	55	75	115	170	148	145	159	98	101	95	50	58	20
2		-	43	115	165	155	170	198	142	138	115	75	105	75
3			-	80	125	120	142	181	135	123	91	64	106	95
4				-	56	41	72	124	102	80	40	66	100	124
5					-	41	85	145	145	120	88	121	151	178
6						-	45	105	105	80	55	96	120	153
7							-	60	75	53	53	98	103	145
8								-	63	60	91	123	104	150
9									-	25	60	72	40	88
10										-	40	65	50	95
11											-	45	64	98
12												-	46	58
13													-	47
14														-

Table 5.2. Pick-up and delivery demands for the customer nodes.

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Sum
Delivery	0	20	25	15	40	20	10	30	30	25	20	15	20	20	290
Pick-up	0	30	30	15	30	15	15	20	35	10	15	15	30	35	295

The TSP solution for this network is 1 – 2 – 3 – 12 – 11 – 4 – 5 – 6 – 7 – 8 – 10 – 9 – 13 – 14 – 1 with cost 641. However, if the vehicle's capacity is taken to be 300 units, this is an impossible sequence to use, since the load value exceeds the capacity both at node 2 and node 14. Note that the cumulative pick-up and delivery demands are within the capacity. Hence the route is feasible in these two respects, but not load feasible. It is possible to show that the cheapest Hamiltonian cycle that is feasible in all three respects is 1 – 13 – 10 – 9 – 8 – 7 – 6 – 5 – 4 – 11 – 12 – 3 – 2 – 14 – 1 with cost 685. Now, let us turn to a lasso solution for the same problem. We assume that there will be no spoke nodes (apart from the depot). Using the model in Halskau, (2000) gives the following optimal solution where node no. 14 turns out to be the junction: 1 – 14 – 2 – 3 – 12 – 11 – 4 – 5 – 6 – 7 – 8 – 10 – 9 – 13 – 14 – 1 with cost 681, which is better than the cheapest feasible Hamiltonian cycle. Further it is easily checked that the above lasso solution is feasible for the given delivery and pick-up demands whatever direction the sequence is performed. The conclusion is that in this respect, relaxing the premise that every customer shall be visited only once can give a better result than insisting on a cycle solution.

Now, applying the lasso construction heuristic from sub-section 4.2 we get the following lasso: 1 – 14 – 13 – 9 – 10 – 11 – 4 – 6 – 5 – 7 – 8 – 12 – 3 – 2 – 14 – 1 with cost 764. The junction is still node 14 as in the optimal solution above, but the sequencing among the loop nodes is quite different. The nearest neighbour part of this heuristic will first create the path 1 – 14. This violates the vehicle's capacity since the load then becomes 305. Hence we change the first node – node 14 into a spoke node and reduce the load value with 35 units, the pick up demand for node 14. Continuing the from node 14 we get 13 – 9 – 10 – 11 – 4 – 6 – 5 – 7 – 8 – 12 – 3. Finally node 2, the last node, is added to the path and we return from node 2 to node 14, changing the status of node 14 from a spoke node to a junction node.

Applying the lasso improvement heuristic, we start with the optimal Hamiltonian cycle given above as the input cycle in step 0. This cycle is not feasible. Starting from the depot the capacity will be violated already at node 2. Changing the status of this node to a (potential) spoke node, we reduce the vehicle load with the pick-up demand for this node. Having done this the rest of the cycle can be served and we return to node 2, changing the status to a junction node. Hence we have the lasso 1 – 2 – 3 – 12 – 11 – 4 – 5 – 6 – 7 – 8 – 10 – 9 – 13 – 14 – 2 – 1 with cost 751. If one traverses the Hamiltonian cycle in the opposite direction, and letting node 14 be the first node after the depot one ends up with the optimal lasso above.

The next examples are dealing with a multiple vehicle problem. Using the same graph and demands as before, we now assume that we have a homogenous fleet of vehicles with capacities equal to 150 units. Hence we know that we need two vehicles or more to handle the planning situation.

We first illustrate the heuristic from sub-section 5.2 using the nearest neighbour heuristic to create sub cycles that are both delivery and pick-up feasible. This gives the following three sub-cycles:

Route 1: 1 – 14 – 13 – 9 – 10 – 11 – 4 – 1 with cost 327 and L(1) = 130units

Route 2: 1 – 12 – 3 – 9 – 2 – 11 – 6 – 5 – 7 – 1 with cost 583 and L(1) = 130units

Route 3: 1 – 8 – 1 with cost 318 and L(1) = 30units.

It is easily checked that all three sub-cycles are load feasible, route 1 taken in the opposite direction, and can then be used for the pick-up and delivery problem. The total cost is 1228.

In the last example we will use the FJ approach. We get the two clusters $\{1,2,5,6,10,13,14\}$ and $\{1,3,4,7,8,9,11,12\}$. By construction, these two clusters are both pick-up feasible and delivery feasible with L(1) = 145units in both cases. Making sub-cycles from the first cluster, the cheapest load feasible Hamiltonian cycle is 1 – 5 – 6 – 10 – 13 – 14 – 2 – 1 with cost 518. However, the lasso 1 – 14 – 13 – 10 – 6 – 5 – 2 – 14 – 1 is load feasible as well, but the cost is only 498. For the last cluster the Hamiltonian cycle 1 – 3 – 4 – 11 – 7 – 8 – 9 – 12 – 1 is load-feasible with cost 493. Note that the feasible route configuration consists of one lasso and one sub-cycle. The total cost becomes 991.

6. CONCLUSIONS AND FURTHER RESEARCH

In this paper we have shown that the pick-up and delivery problem is substantial more complex than the traditional pure VRP – even when restricted to the case when all demands either originate at the depot or must be brought back to the depot. The paper also shows that three kinds of feasible criteria must be obeyed and that load feasibility severely restricts the applications of traditional heuristics for TSP and vehicle routing. Further, the examples show that the introduction of lassos can give better solutions to the planning problem than solutions based exclusively on cycles. In addition lasso solutions make the routes more flexible both in terms of changing demands and free space on the truck. However, our analysis is preliminary at this stage and the problems tested have been small, but this approach seems to compare favourably with the classical approach in terms of total distance. To the best of our knowledge no exact model for multi-vehicle lasso problems is described in the literature. Such models can probably be formulated in several different ways and it will be of importance to formulate them, test and compare how they behave with real life in-put data. This paper represents a preliminary examination of the new alternative approach. There is much additional algorithmic work that needs to be done.

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