EVOLUTIONARY STRATEGIES FOR SOLVING FRUSTRATED PROBLEMS

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Abstract

The main elementary processes and strategies of evolution are investigated and described by simple mathematical models (stochastic networks). Special attention is devoted to FISHER-EIGEN *type models as well as to* BOLTZMANN-, DARWIN- *and* HAECKEL- *strategies modelling basic elements of frustrated problems in biological evolution respectively. Several applications of evolutionary strategies to frustrated optimization problems are discussed, in particular the evolution of complex strings satisfying contradictory conditions and the optimization of a network of streets connecting a random distribution of points.*

I. The main strategies of evolution

Analyzing the mechanisms of natural evolution we find several basic strategies [1, 2, 3]:

Boltzmann strategy: One fundamental goal nature is the optimization of certain thermodynamic functions. The BOLTZMANN strategy has three important elements:

1. Motion along gradients to reach steepest ascent of entropy
2. Various stochastic processes including thermal and hydrodynamic fluctuations which avoid locking.
3. Decrease in temperature during the adiabatic expansion.

Darwin strategy: This second important natural strategy appears in the universe only in the process of biogenesis. The basic elements of a DARWIN strategy are:

1. Self-reproduction of good species with maximal fitness.
2. Mutation processes due to error reproductions.
3. Increase of the precision of self reproduction.

Haeckel strategy: With increasing complexity the cell organisms developed a life-cycle consisting of several periods as youth, period of growth and learning, period of self-reproduction and death. We incorporate further in this strategy the mating (sexual reproduction). The result of mating is a new individual which is a combination of its parents.

II. Models of evolution processes

Let as consider a set of species which are characterized by a set of strings, lists or matrices playing here the role of the "genotypes". All possible objects may be considered as elements of an abstract, metric space, the genotype space $\mathbb{G}$. We assume that each genotype is connected with a set of properties forming the "phenotype"; in this way we introduce also a phenotype space $\mathbb{Q}$. The phenotype is valuated in the evolution. Mathematically this means that any element i is associated with a set of real number E_i. Our assumption is, that for each object i an occupation number N_i (stochastic picture) or fraction x_i is defined. Here N_i denotes the number of representatives of the objects of kind i in the system and x_i a corresponding fraction. The most simple model of an evolutionary dynamics is the FISHER-EIGEN model which is based on the assumption that the competing objects $i = 1, 2, ..., s$ have different reproduction rates $E_1, E_2, ..., E_i, ..., E_s$. These scalar quantities play now the role of the values. The dynamics of the fractions is given by the differential equations [4].

$$\frac{d}{dt}\, x_i = (\, E_i - k(t)\,)\, x_i(t) \qquad (1)$$

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The FISHER-EIGEN model is the simplest of all models of competition. It refers to an oversimplified case since there is no real interaction between the species. More realistic models take the coupling between the competing objects into account [5].

A generalization of (1) takes into account that the reproduction rates as well as the death rate (and possibly also the other values) depend on the age of the individuals belonging to the species i [6, 7]. The stochastic network model used in this work is a generalization of EIGENs deterministic equations [7]. We consider N competing objects (N fixed in time) which are subject to self-replication, death, mutation and selection. Let N_i be the number of objects of type i. The elementary transition is the one–step process

$$N_i \longrightarrow N_i + 1;\ N_j \longrightarrow N_j - 1 \tag{2}$$

Two kinds of such transition are assumed. The first is a simple change of type (modelling mutation) occurring with the transition probability

$$W(N_i,\, N_j - 1\,|\,N_i,\, N_j) = A_{ij}(\Delta_{ij},\, \tau_j)\, N_i \tag{3}$$

Here τ_j is the individual age of the object j after appearance in the game and $\Delta_{ij} = E_i - E_j$ the difference of the values of the target i and the source j. A reasonable assumption in the spirit of annealing strategies is

$$A_{ij} = A(\tau_j) \left\{ \begin{array}{rcl} 1 & : & \Delta_{ij} \geq 0 \\ exp(-\frac{\Delta_{ij}}{T} & : & \Delta_{ij} < 0 \end{array} \right. \tag{4}$$

Here $A(\tau)$ is simple non–negative function which has a maximum at finite age and goes to zero for $\tau \rightarrow \infty$.

The second kind of translation occurs after comparison of two objects i and j. If object i has a higher value, then object j is replaced by object i with the transition probability

$$W(N_i,\, N_j - 1\,|\,N_i,\, N_j) = \frac{1}{N}\, B_{ij}\, N_i\, N_j \tag{5}$$

A reasonable choice for the matrix is for $\Delta_{ij} > 0$

$$B_{ij} = B(\tau_i)\, \Delta_{ij} \tag{6}$$

for $\tau \rightarrow \infty$, $B(\tau)$ is an increasing function which converges for $\tau >> \tau_0$ to a constant.

This assumption includes that rate of this kind of transitions is proportional to the improvement and further that objects younger than τ_0 are not subject to competition. The model described above is surprisingly general and includes the EIGEN model as well as annealing models as special cases.

III. APPLICATIONS TO THE OPTIMIZATION OF STRINGS AND STREET NETWORKS

In the first application we considered the evolution of strings consisting of four kinds of letters **A, B, C, D**. A valuation of the strings s_i

$$E = E(s_i) \tag{7}$$

was introduced by the following simple rules

$$E(\mathbf{A}) = E(\mathbf{B}) = E(\mathbf{C}) = E(\mathbf{D}) = 1 \tag{8}$$

$$E = \left\{ \begin{array}{lcl} E + a & : & \mathrm{MID}(s_i,p,2)\in\{\mathrm{AB,BC,CD,DA}\} \\ E + b & : & \mathrm{MID}(s_i,p,1)=\mathrm{MID}(s_i,p+g,1) \end{array} \right. \tag{9}$$

The first rule (9) favours alphabetic order **"ABCDABCDABC..."** which leads to the second periodic rule with the period 4. This favours periodic repetitions with the period g. If $g \neq 4$ then the tendencies to generate strings with periodicity 4 or g are contradictory, i.e. the system is frustrated.

Our simulations show the Selforganization of structures with a typical long range order showing a correlation length which is as large as the string length l. Our game is a stochastic one and there are permanently N strings participating in it ($N = 2 \ldots 32$). Good strings have the chance to make "offspring" which in our game are identical or slightly modified strings. We have used the following types of mutations:

1. point mutations, i.e. changing one position in the string,
2. associations , i.e. addition of one are two letters to the right or to the left of the string,
3. recombination, i.e. making a new string from the left end of one member of the ensemble and the right end of another member of the ensemble.

Any new string which appeared in the game at the time t_n was associated with the age $\tau = t - t_n$. τ_0 denotes the time of maturity. In the game we have assumed that the mutation rates $m(\tau)$ are decreasing with the age and tend to zero for $\tau > \tau_0$. In the opposite the reproduction and selection rate $r(\tau)$ were assumed to be small in the youth $\tau < \tau_0$ and to converge to a constant value at $\tau > \tau_0$. We have made many simulations using several sets of parameters. The present model may be considered as a prototype of realistic models of the evolution of biomolecules.

In our second application we simulated the optimization of street networks. As a concrete example we apply evolutionary search to a street network. A street network is a graph g connecting a given set of points $\mathrm{p}_1, .., \mathrm{p}_n$ in the plane. The edges of the graph g correspond to streets or highways the nodes to houses or towns. There are two obvious factors to be minimized:

1.) The distance from each point to each other point shall be as small as possible.
2) The network shall be as short as possible to minimize the cost of construction.
Thus the functional V we intend to optimize consists of two parts:

$$\mathrm{V}(g) = (1-\lambda)\mu(g) + \lambda \mathrm{L}(g). \tag{10}$$

The first part μ is the mean of the length of the shortest path from point i to point j averaged over all pairs (i,j)

$$\mu = \frac{1}{n(n-1)} \sum_{i<j} \mathrm{l}(\gamma_{\min}(i,j)) \tag{11}$$

with $l(\gamma_{\min}(i,j))$ being the the length of the shortest path $\gamma_{\min}(i,j)$ from i to j. The second part L is simply the length of the whole network.

The objects of evolutionary search are all connected graphs g on the set of points $\mathrm{p}_1,..,\mathrm{p}_n$ the number is of order $2^{n(n-1)/2}$. The parameter λ allows us to weighten the importance of short path or construction cost respectively. If building routes is cheap we have a small λ resulting in a dense graph (see fig 1) which approaches the fully coupled graph for $\lambda \rightarrow 0$. If on the other hand construction costs are very high the resulting graph is only sparsely coupled and approximates the minimum spanning tree in the limit $\lambda \rightarrow 1$.

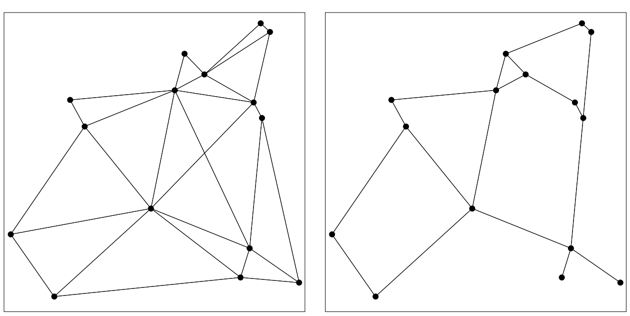

Figure 1: Right: randomly coupled initial graph $\lambda = 0.005$. Left: optimized graph $\lambda = 0.05$. The number of points is $n = 15$.

IV. Discussion

The number of representatives (objects) in the ensemble was varied between 2 and 32 . We have calculated the mean value, the dispersion and the best results. It was shown that the proposed algorithm gives reasonable results. We find significant improvements by including Darwinian and Haeckelian elements into the search strategy [4-6]. Boltzmann, Darwin and Haeckel strategies show several parallels but also essential differences [8]. All these strategies are well suited to find the extrema in landscapes of potential functions. In general it will depend on the structure of this landscape, what search strategy is the better one. The qualitative analysis carried out in this work suggests that in the case that no knowledge about the structure of the landscape is available, it will be advantageous to apply the Boltzmann strategy combined with annealing. This strategy seems to be more universal; it will always work. However, thermodynamic processes have the tendency to be locked in relative extrema surrounded by high thresholds. On the other hand Darwinian and Haeckelian processes are able to cross high barriers by tunneling if the next minimum is close.

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