Empirical Computation of the Quasi-Optimal Number of Informants in Particle Swarm Optimization

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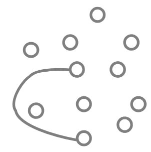
Introduction

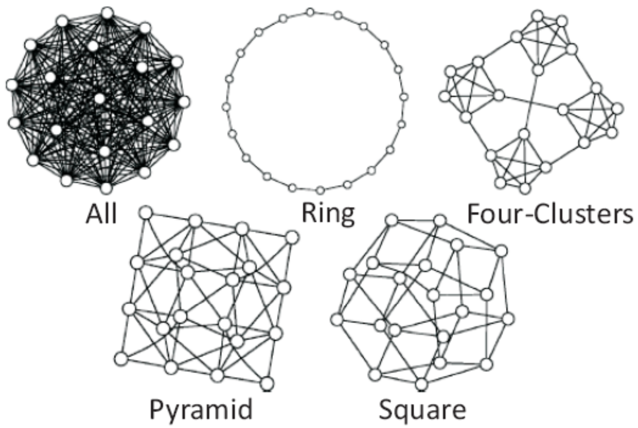
- In Canonical and Standards (2006, 2007, 2011) versions of PSO, the calculation of a new particle's velocity (and hence the particle's position) is influenced by just two informant terms: the particle's best previous location, and the best previous location of any of its neighbors

$$\mathbf{v}_i^{t+1} = \chi\left(\mathbf{v}_i^t + U^t[0,\varphi_1]\cdot(\mathbf{p}_i^t - \mathbf{x}_i^t) + U^t[0,\varphi_2]\cdot(\mathbf{b}_i^t - \mathbf{x}_i^t)\right)$$



- Mendes et all. 2004 proposed the Fully Informed PSO (FIPS): particle's velocity can be adjusted by any number of terms, since important information given by other neighbors may be neglected through overemphasis of on the single best neighbor.

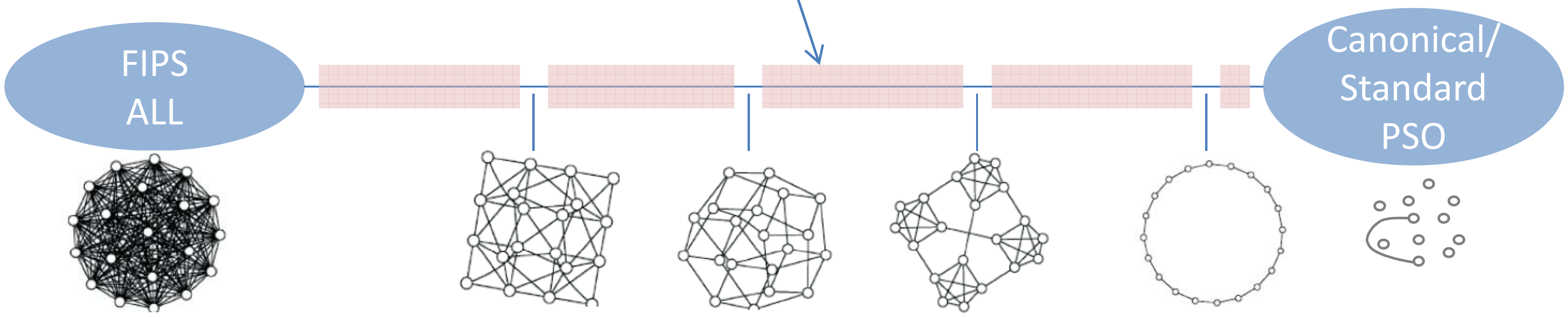
$$\mathbf{v}_i^{t+1} = \chi\left[\mathbf{v}_i^t + \sum_{j\in\mathcal{N}_i} U^t\,[0,\varphi_j]\cdot(\mathbf{p}_j^t - \mathbf{x}_i^t)\right]$$



- In FIPS, the neighborhood of informants is arranged in structured topologies

The quest for an optimal number of informants

- Again, important information may be neglected through overemphasis, in this case, by structured sets of neighbors



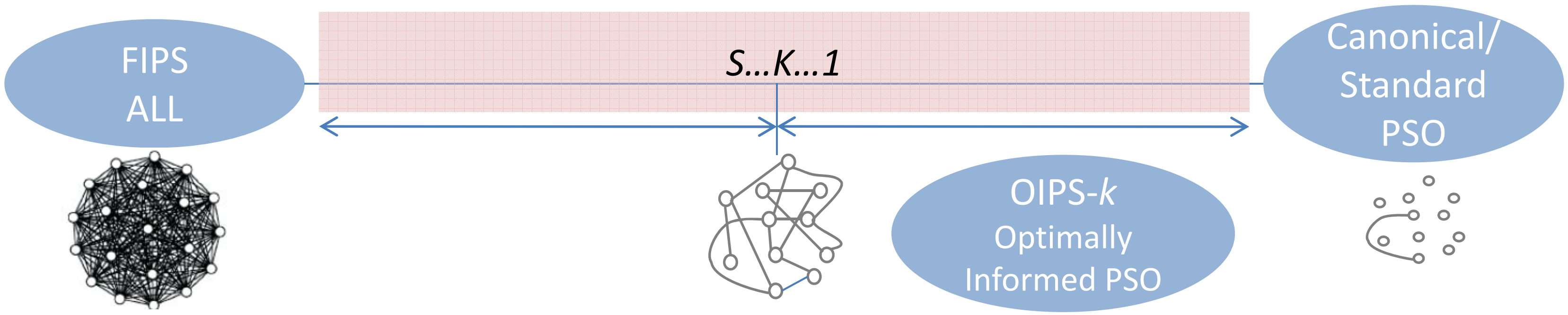
- Motivation: generalize the number of neighbors that inform particles, in order to discover whether there exists a quasi-optimal number of informants for a particular problem

Hypothesis:

certain numbers (sets) of informant neighbors may provide new essential information about the search process, hence leading the PSO to perform more accurately than existing versions

The quest for an optimal number of informants

- Generalization of the number of informant terms from 1 to *S* (swarm size), resulting *S* different versions of PSO, each one of them with neighborhoods containing *k* informant particles (in FIPS-ALL, *S*=*k*)



- Providing each *k* neighborhood with structured topologies is impracticable (enormous number of graphs combinations)

- We simply select *k* random (uniformly) neighbors in the swarm, for each particle *I* and each time step *t* (topology independence analysis)

$$\mathcal{N}_i^t = \{n_1, \ldots, n_k\} \mid \mathcal{N}_i^t \subset S^t, \forall n_j, n_h \in \mathcal{N}_i^t, n_h \neq n_j \neq i$$

The quest for an optimal number of informants

- Pseudocode of OIPS-*k*

for each *k* = *{1..S}*, a different algorithm can be developed

$t \leftarrow 0$
$\varphi_j = \varphi / k$
initialize(S^t) /* Swarm S^0 with N particles */
while $t < MAXIMUM_t$) **do**
  for each particle i^t of the swarm S^t **do**
    $\mathcal{N}_i^t = generate\_neighborhood(k, i, S^t)$
    $\mathbf{v}_i^{t+1} = update\_velocity(\mathbf{v}_i^t, \mathbf{x}_i^t, \varphi_j, \mathcal{N}_i^t)$
    $\mathbf{x}_i^{t+1} = update\_positon(\mathbf{x}_i^t, \mathbf{v}_i^{t+1})$
    $\mathbf{p}_i^{t+1} = update\_local\_best(\mathbf{p}_i^t, \mathbf{x}_i^{t+1})$
  end for
  $t \leftarrow t + 1$
end while
Output: b /*The best solution found*/

Neighborhood generation with *K* informant neighbors

$$\mathcal{N}_i^t = \{n_1, \ldots, n_k\} \mid \mathcal{N}_i^t \subset S^t, \forall n_j, n_h \in \mathcal{N}_i^t, n_h \neq n_j \neq i$$

Full informed velocity calculation

$$\mathbf{v}_i^{t+1} = \chi \left[\mathbf{v}_i^t + \sum_{j \in \mathcal{N}_i} U^t\,[0, \varphi_j] \cdot (\mathbf{p}_j^t - \mathbf{x}_i^t) \right]$$

Experimental setup

- Evaluation of all possible versions of OIPS-*k* (with *k={1..S}* and *S=30*)

Implementation in C++ MALLBA Library [online available]

- Experimental framework proposed in *S.S. Real-parameter optimization CEC'2005* (problem dimension 30 continuous variables)

f	Name	Intervals	f^*
f1	Shifted Sphere	[-100, 100]	-450
f2	Shifted Schwefel 1.2	[-100, 100]	-450
f3	Shifted Rotated High Conditioned Elliptic	[-100, 100]	-450
f4	Shifted Schwefel's Problem 1.2 with Noise	[-100, 100]	-450
f5	Schwefel's Problem 2.6	[-100, 100]	-310
f6	Shifted Rosenbrock's	[-100, 100]	390
f7	Shifted Rotated Griewank's. Global Optimum Outside of Bounds	[0, 600]	-180
f8	Shifted Rotated Ackley's with Optimum on Bounds	[-32, 32]	-140
f9	Shifted Rastrigin's	[-5, 5]	-330
f10	Shifted Rotated Rastrigin's	[-5, 5]	-330
f11	Shifted Rotated Weierstrass	[-0.5, 0.5]	90
f12	Schwefel's Problem 2.13	[$-\pi$, π]	-460
f13	Shifted Expanded Griewank's plus Rosenbrock's	[-3, 1]	-130
f14	Shifted Rotated Expanded Scaffer's F6	[-100, 100]	-300
f15	Hybrid Composition (f1-f2,f3-f4,f5-f6,f7-f8,f9-f10)	[-5, 5]	120
f16	Rotated Version of Hybrid Composition f15	[-5, 5]	120
f17	F16 with Noise in Fitness	[-5, 5]	120
f18	Rot. Hybr. Comp. (f1-f2,f3-f4,f5-f6,f7-f8,f9-f10)	[-5, 5]	10
f19	Rot. Hybr. Comp. Narrow Basin Global Optimum	[-5, 5]	10
f20	Rot. Hybr. Comp. Global Optimum on Bounds	[-5, 5]	10
f21	Rot. Hybr. Comp. (f1-f2,f3-f4,f5-f6,f7-f8,f9-f10)	[-5, 5]	360
f22	Rot. Hybr. Comp. High Condition Number Matrix	[-5, 5]	360
f23	Non-Continuous Rotated Hybrid Composition	[-5, 5]	360
f24	Rot. Hybr. Comp. (f1,f2,f3,f4,f5,f6,f7,f8,f9,f10)	[-5, 5]	260
f25	Rot. Hybr. Comp. Global Optimum Outside of Bounds	[2, 5]	260

CEC'2005 Benchmark:
25 problem functions: unimodal, multimodal, rotated, shifted, expanded, hybrid composed

Problem Dimension D: 30
Fitness Evaluations FE: Dx10,000

OIPS-k Parameters

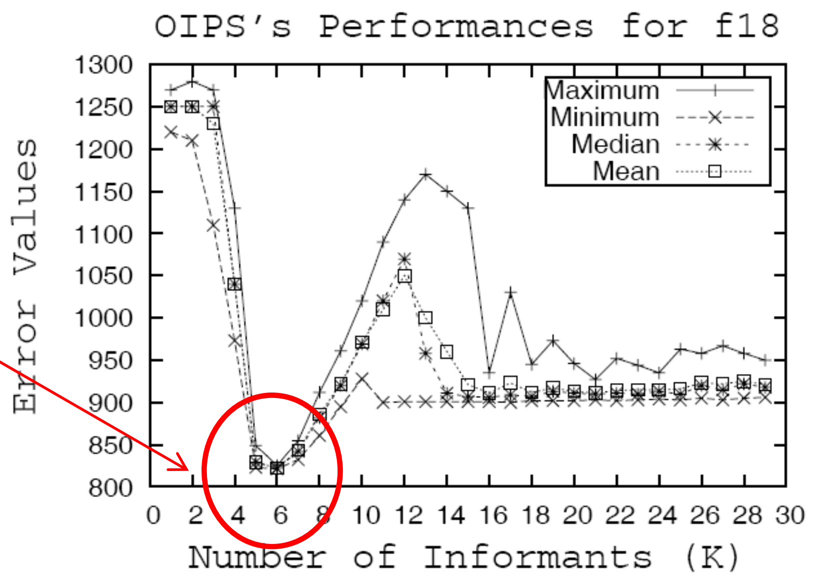
Swarm size	Ss	30
Acceleration coefficient	φ	4.1
Constriction coefficient	χ	0.7298

Understanding the research

- Experimental phase
  - 30 OIPS-k versions (k=1..30)
  - 25 Benchmark functions (CEC'2005)
  - 25 Independent runs
  - A total number of 18,750 (30x25x50) experiments
  - Statistical analysis (Friedman's and Holm's tests)

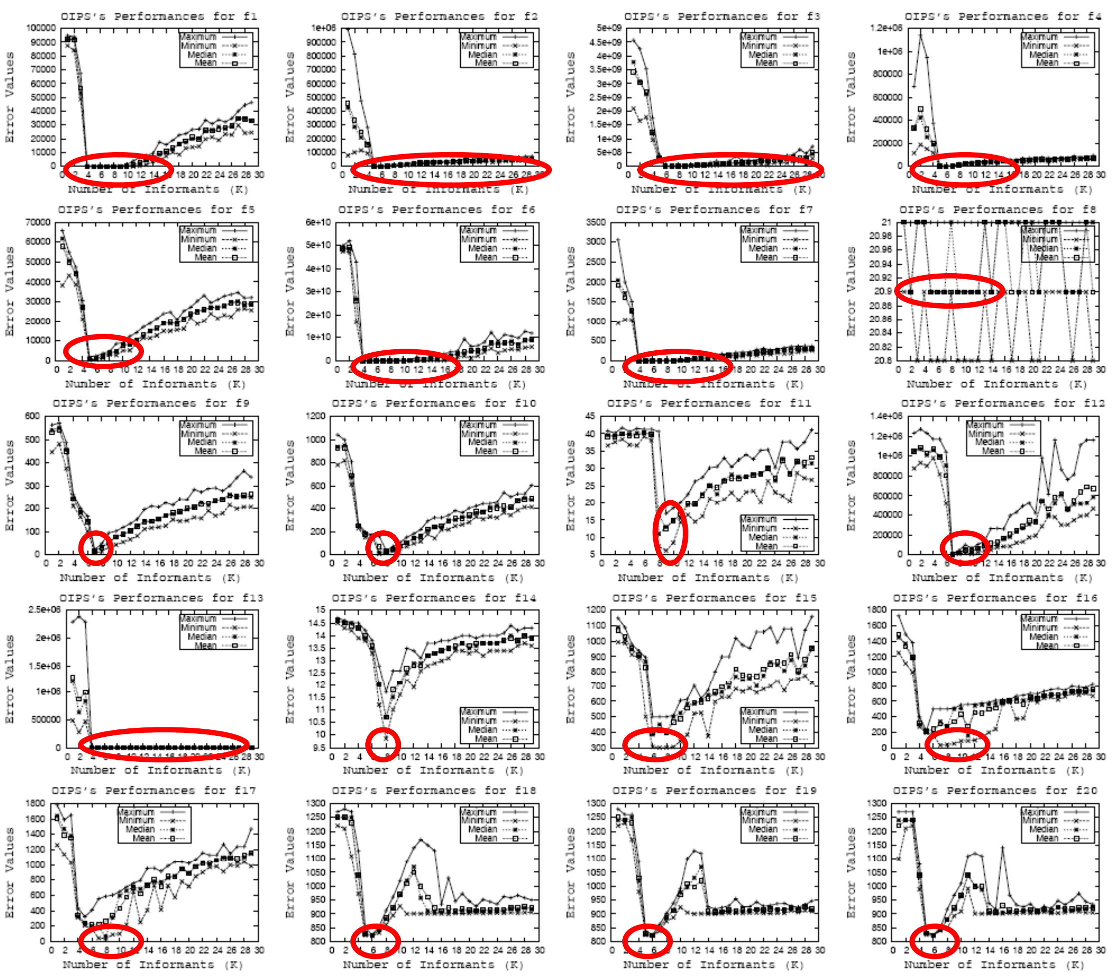
- For each problem function: the maximum, median, mean, and minimum error fitness are plotted

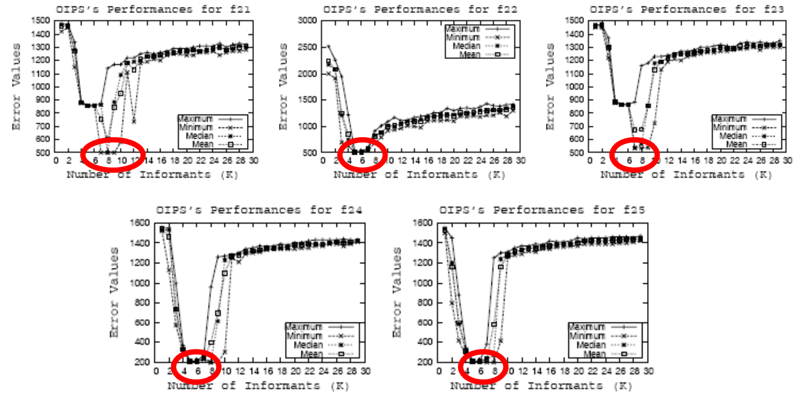
Best performing OIPS-k=6

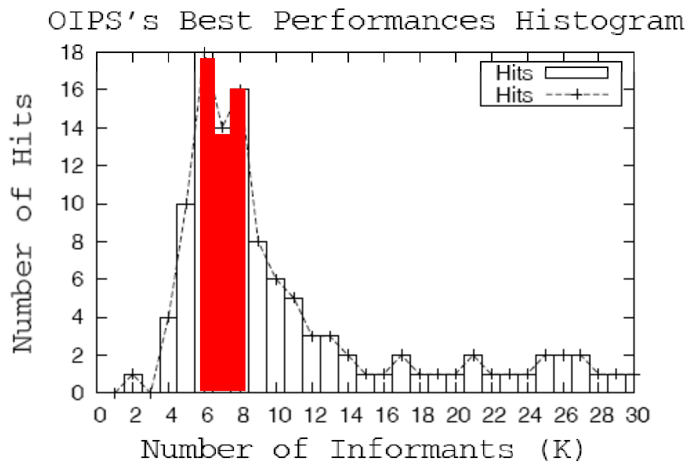


Understanding the research: Impact of the number of Informants

- For all the CEC'2005 functions

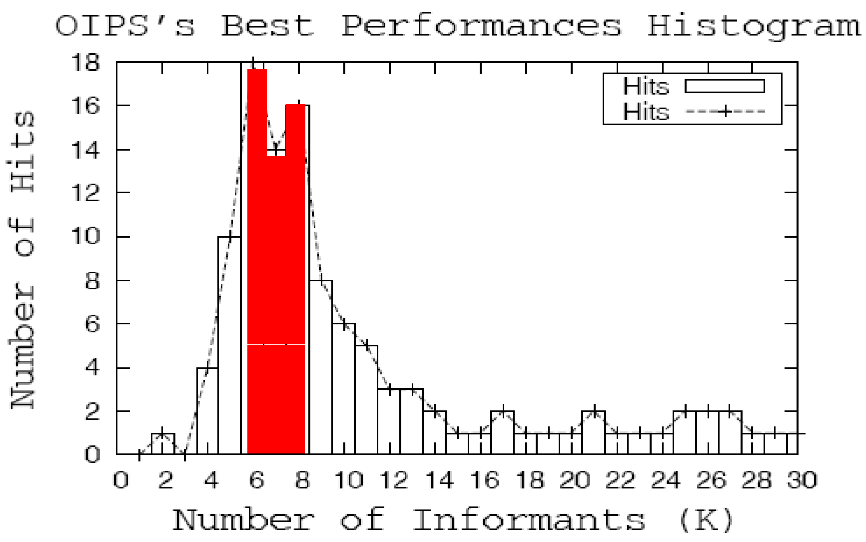






Understanding the research: Impact of the number of Informants

- Observations and implications
  - OIPS-6, the most promising version of PSO (with k=6 informants)
  - The interval between 5 and 8 informants concentrates most of successful runs
  - A number of 8 informants is also appropriate.
  - Combining 6 and 8 informants could be a source of new competitive algorithms
  - There are sets of functions that share close curve shapes.
  - In fact, biased functions to the same optimum share similar curve shapes. ¿Is it because an unknown feature of CEC'2005 functions?



Understanding the research: Performance comparisons

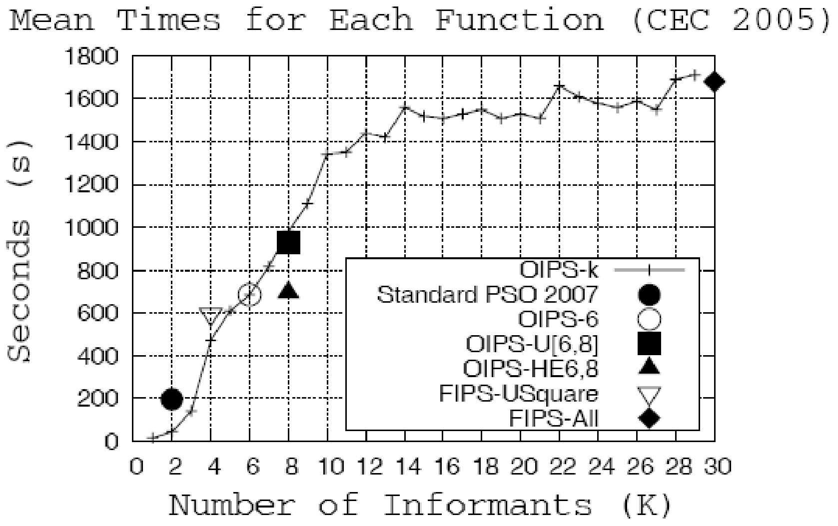
- The best OIPS-k (and its combinations) against FIPS-Usquare (the best one in Mendes et al. 2004), FIPS-ALL, and Standard PSO 2007

Algorithm	Best performance in functions (CEC'2005)	Number of functions	Statistical Ranking (Friedman)
OIPS-HE{6,8}	**f1, f5, f7, f9, f18, f19, f20, f22, f24, f25**	**10**	**2.58**
OIPS-6	f1, f2, f3, f6, f7, f19, f20, f24, f25	9	2.86
FIPS-Usquare	f1, f3, f6, f10, f12, f13, f15, f16, f17	9	2.88
OIPS-U[6,8]	f1, f14, f21, f23	4	3.26
FIPS-ALL	f11	1	3.76
Standard PSO 2007	f8	1	5.66

(Two new combinations of OIPS-6 and OIPS-8: OIPS-HE{6,8} and OIPS-U[6,8])

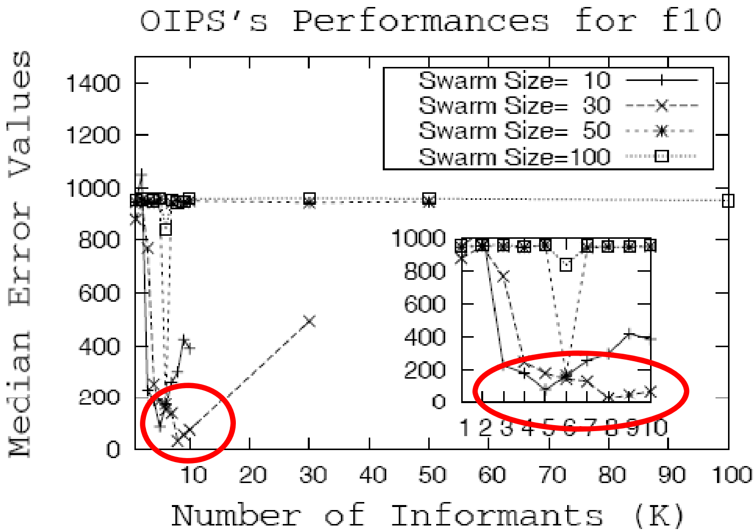
Understanding the research: Computational effort

- Computational effort
  - Mean running times in which all the versions of OIPS-*k*, as well as all other compared algorithms, have found the best mean error for all the CEC'2005 functions
  - The running time increases with the number of informants, although it seems to stabilize from OIPS-15 to OIPS-30 (e. g. FIPS-ALL)
  - Almost all the compared algorithms requited similar running times: from 600 to 900 seconds
  - The time the random selection operation spends is not significant with regards to the time of calculating the new velocity

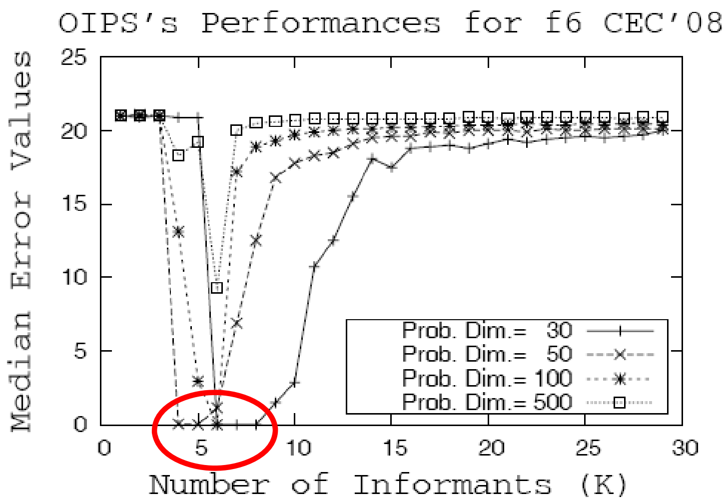


Understanding the research

- Influence of the swarm size
  - Additional configurations of swarm size: 10, 30, 50, and 100
  - Best performance for neighborhoods with *k* between *6 and 9* informants

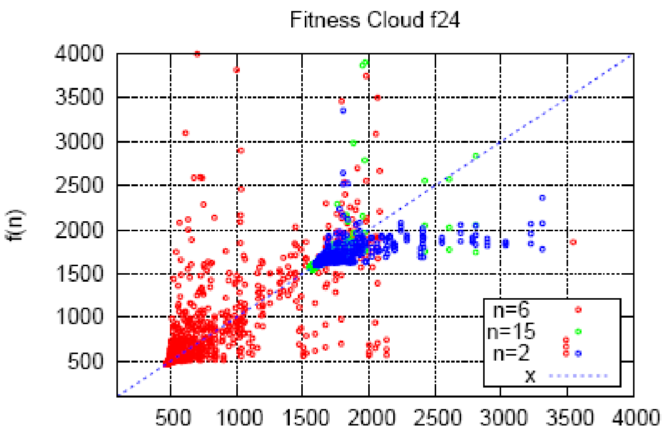


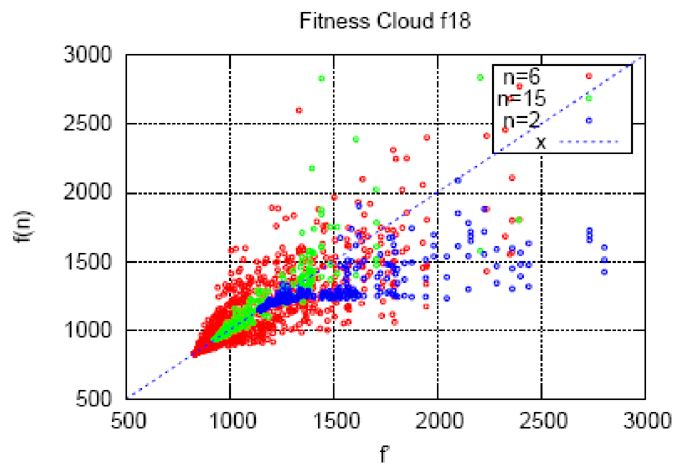
- Influence of the problem dimension
  - Additional experiments in the scope of CEC'2008 and CEC'2010
  - OIPS-6 obtained the best performance for the studied Shifted Ackley's function f6 in CEC'2008 (f10 in CEC'2010). Similar curve shapes to CEC'2005 functions



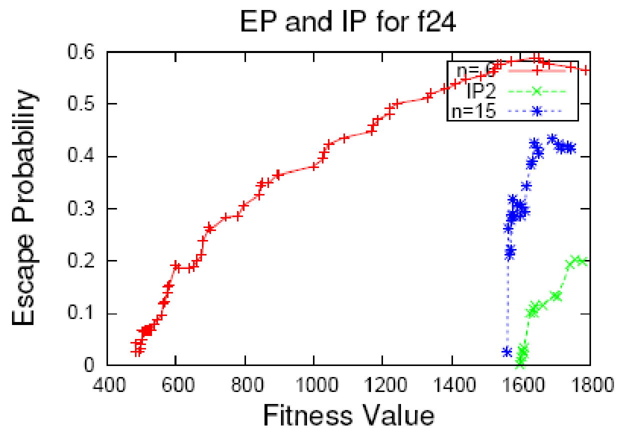
Further Analysis: Evolvavility

- Fitness clouds: comparing OIPS-6, OPIS-2 (Standard), and OIPS-15





- Escape probability



Conclusions and future work

- We have proposed a new version of Informed PSO, called OIPS-k with the possibility of managing any neighborhood size k, from 1 informant to all of them in the swarm (FIPS-ALL)

- After the experimentation we conclude:
  - A number of 6 informants makes the algorithm to perform with high success
  - Performance comparisons against other techniques lead us to propose our OIPS-*k*
  - The interval between 5 and 10 informants concentrates most of the successful runs
  - CEC'2005 functions biased to the same optimum share similar curve shapes of OIPS-*k*'s performances
  - The highest the number of informants, the longer the running time
  - Similar behavior observed in our experiments independently of the swarm size and the problem dimension

Conclusions and future work

- Future work:
  - Investigating other elemental features of the PSO
  - Applying new concepts of the Standard PSO 2011 to informed versions
  - Analytical investigations on the success of 6-8 informants
  - Experimentation with other current benchmarks (CEC’2008, BBOB, CEC’2010, …)

Questions!!!