On the Scalability of Multi-objective Metaheuristics for the Software Scheduling Problem

NEO Networking and Emerging Optimization

LENGUAJES Y CIENCIAS DE LA COMPUTACIÓN UNIVERSIDAD DE MÁLAGA

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GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN
F.E.D.E.R.

| M*: Multidisciplinar Multiobjective Metaheuristics
| TIN2008-06491-C04-01 Ministerio de Ciencia e Innovación (Spain)
| UMA - University of Málaga

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Introduction

- Current software projects are very complex
- They can involve hundreds of people and tasks
- An efficient way of assigning employees to tasks is required
- An automatic software tool can assist to the software project manager
- Problem: assign employees to tasks with a given dedication degree

Employee

Salary

Maximum dedication

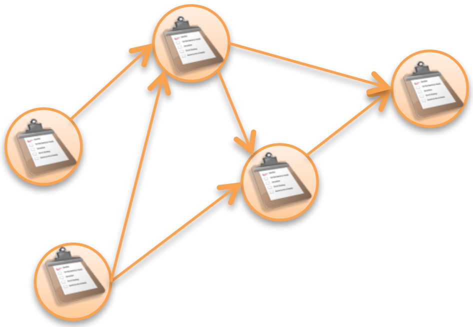
Skills

Task

Effort

Required skills

TPG



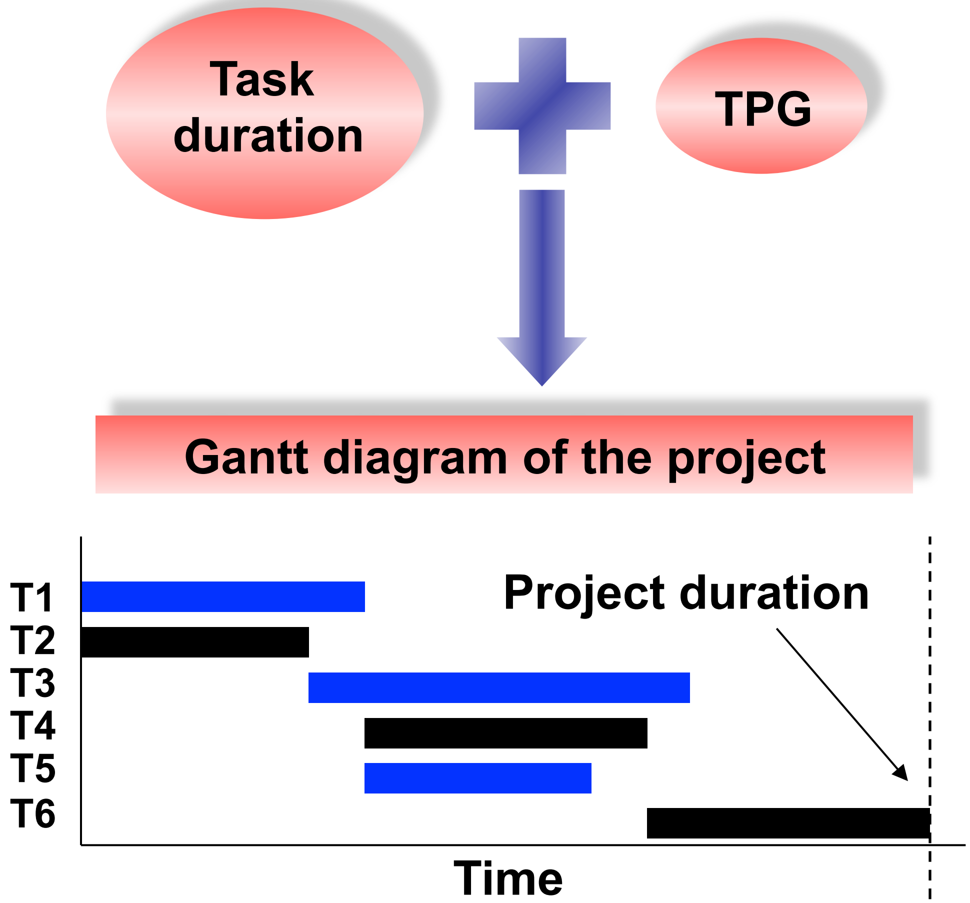
- What is the performance of metaheuristics when the problem size increases?

Problem Formulation: duration

- Project duration (computation)

	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5	0.7	1.0	0.0
E2	0.0	0.0	0.2	0.1	0.5	0.8
E3	0.2	0.0	0.0	0.6	1.0	1.0
E4	0.4	0.6	0.0	0.0	0.0	1.0
	∑	**0.8**				

$$\frac{\textbf{Effort T2}}{\square} = \textbf{Duration T2}$$



Task duration + **TPG**

Gantt diagram of the project

Problem Formulation: cost

- Project cost (computation)

	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5	0.7	1.0	0.0
E2	0.0	0.0	0.2	Dur. T4	0.5	0.8
E3	0.2	0.0	0.0	× 0.6	1.0	1.0
E4	0.4	0.6	0.0	0.0	0.0	1.0

Time employee E3 spends on task T4

Problem Formulation: cost

- Project cost (computation)

	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5			
E2	Dur. T1 ×	Dur. T2 ×	Dur. T3 ×	Dur. T4 ×	Dur. T5 ×	Dur. T6 ×
E3	0.2	0.0	0.0	0.6	1.0	1.0
E4	0.4	0.6	0.0	0.0	0.0	1.0

$\sum$ = time the employee spends on the project

✖ Salary of E3

Cost of employee E3 due to its participacion

Problem Formulation: cost

- Project cost (computation)

	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5	0.7	1.0	0.0
E2	0.0	0.0	0.2	0.1	0.5	0.8
E3	0.2	0.0	0.0	0.6	1.0	1.0
E4	0.4	0.6	0.0	0.0	0.0	1.0

Cost of employee E1 due to its participation

Cost of employee E2 due to its participation

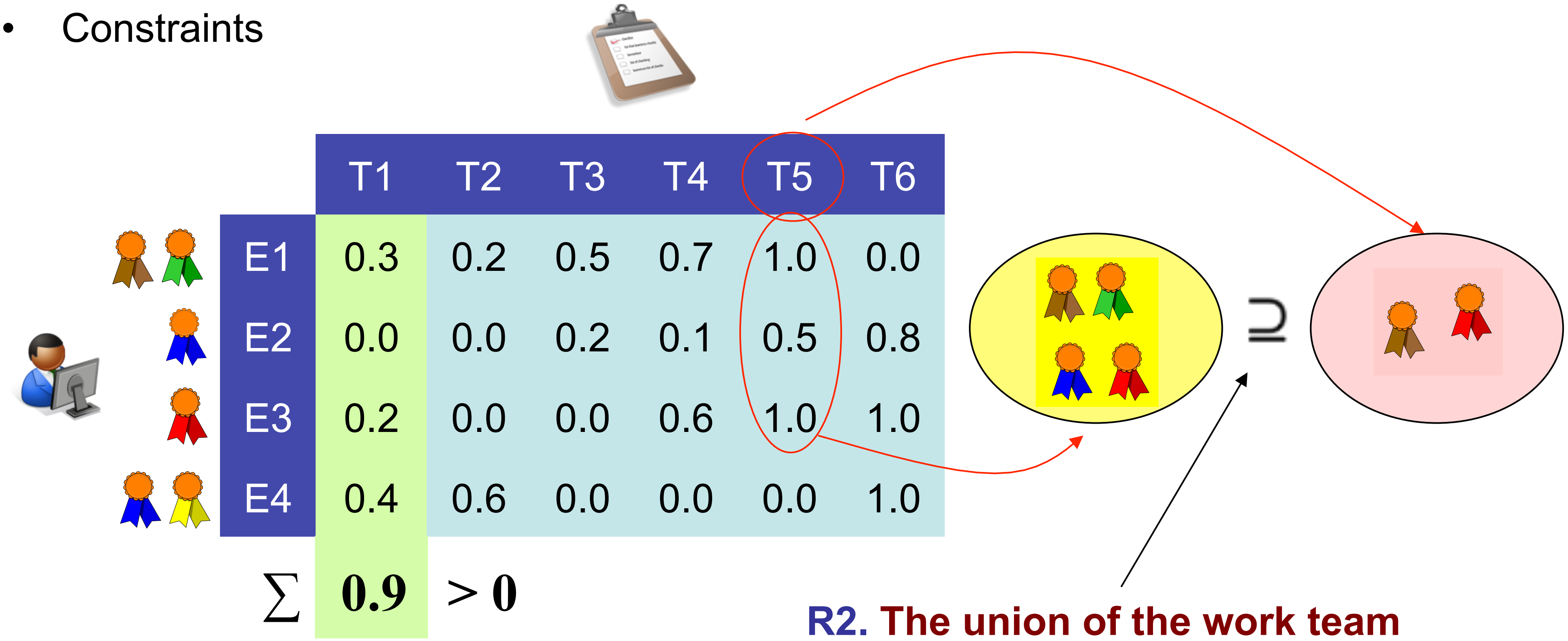
Cost of employee E3 due to its participacion

Cost of employee E4 due to its participacion

$\sum =$ Project cost

Problem Formulation: constraints

- Constraints



	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5	0.7	1.0	0.0
E2	0.0	0.0	0.2	0.1	0.5	0.8
E3	0.2	0.0	0.0	0.6	1.0	1.0
E4	0.4	0.6	0.0	0.0	0.0	1.0

$\sum \mathbf{0.9} > 0$

$\supseteq$

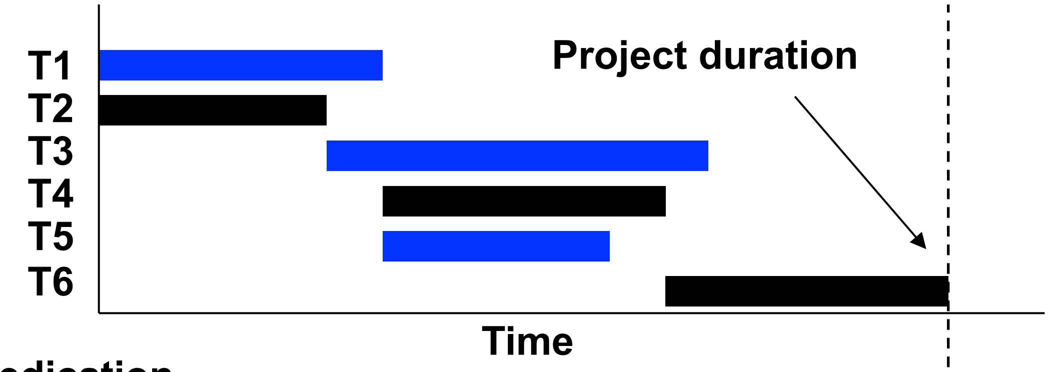
R1. All tasks must be performed

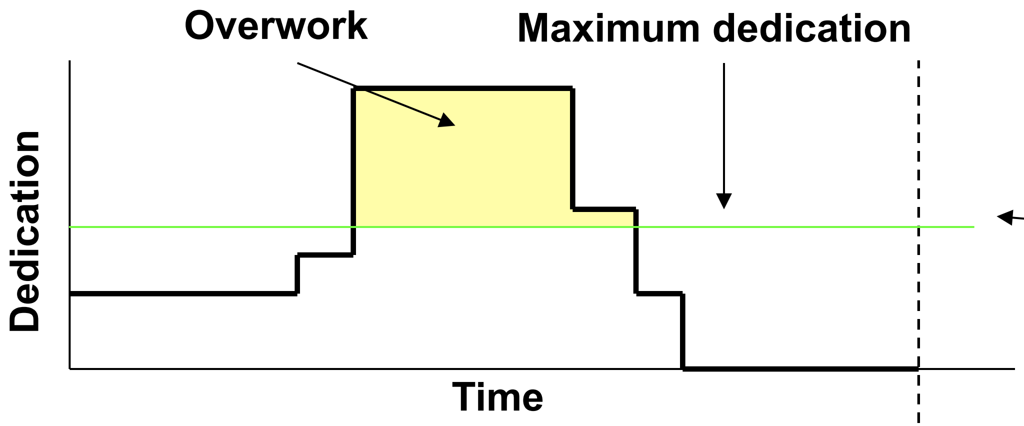
R2. The union of the work team skills must include the required skills of the task they perform

Problem Formulation: constraints

- Constraints (cont.)

	T1	T2	T3	T4	T5	T6
E1	0.3	0.2	0.5	0.7	1.0	0.0





R3. No employee must exceed her/his maximum dedication

Algorithms in the comparison

NSGA-II

- Generational GA
- Ranking & Crowding

PAES

- (1+1) Evolution Strategy + External Archive
- Adaptive Grid

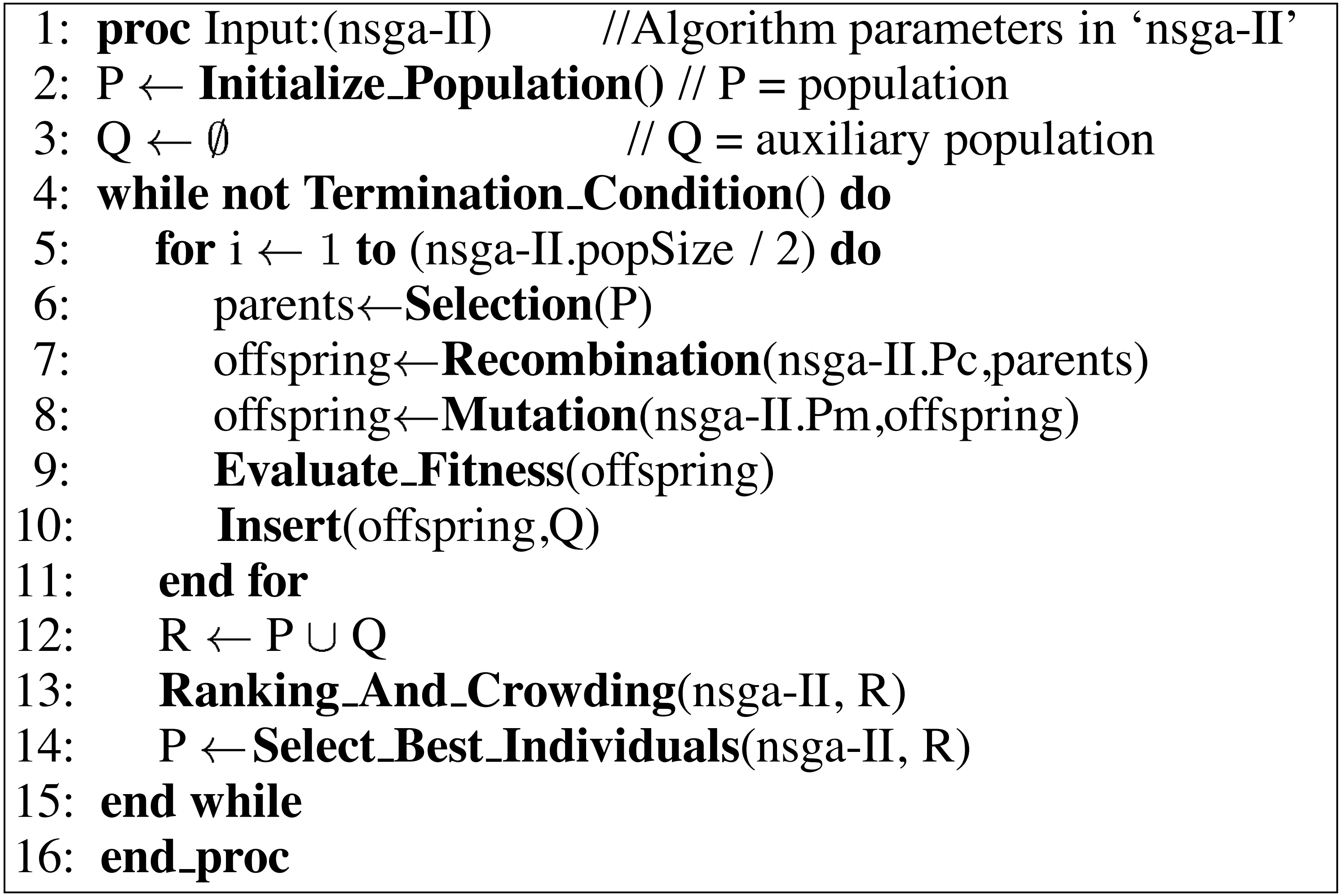
DEPT

- Differential Evolution
- Pareto Tournament

MO-FA

- Firefly Algorithm
- Light intensity & Firefly attraction

Algorithms: NSGA-II



```
proc Input:(nsga-II)        //Algorithm parameters in ‘nsga-II’
P ← Initialize_Population() // P = population
Q ← ∅                       // Q = auxiliary population
while not Termination_Condition() do
    for i ← 1 to (nsga-II.popSize / 2) do
        parents←Selection(P)
        offspring←Recombination(nsga-II.Pc,parents)
        offspring←Mutation(nsga-II.Pm,offspring)
        Evaluate_Fitness(offspring)
        Insert(offspring,Q)
    end for
    R ← P ∪ Q
    Ranking_And_Crowding(nsga-II, R)
    P ←Select_Best_Individuals(nsga-II, R)
end while
end_proc
```

Algorithms: PAES

```
proc Input:(paes)        //Algorithm parameters in ‘paes’
archive ← $\emptyset$
currentSolution ← Create_Solution(paes) // Creates an initial solution
while not Termination_Condition() do
    mutatedSolution←Mutation(currentSolution)
    Evaluate_Fitness(mutatedSolution)
    if IsDominated(currentSolution, mutatedSolution) then
        currentSolution ← mutatedSolution
    else
        if Solutions_Are_Nondominated(currentSolution, mutatedSolution) then
            Insert(archive, mutatedSolution)
            currentSolution ← Select(paes, archive)
        end if
    end if
end while
end_proc
```

Algorithms: DEPT

```
proc Input:(dept)        //Algorithm parameters in 'dept'
P ← Initialize_Population()
while not Termination_Condition() do
    for i ← 1 to dept.popSize do
        Randomly select three different indices $i_1$, $i_2$ and $i_3$
        $v = P[i_1] + \lambda \cdot (Best - P[i]) + F \cdot (P[i_2] - P[i_3])$ // Trial vector
        $u$=Recombine($v$, $P[i]$)
        Evaluate($u$)
        $P[i]$ = Replacement($u$, $P[i]$) // Taking into account Pareto dominance
    end for
end while
end_proc
```

$$MOF(i) = IsDominated(i) * PS + Dominates(i)$$

Algorithms: MO-FA

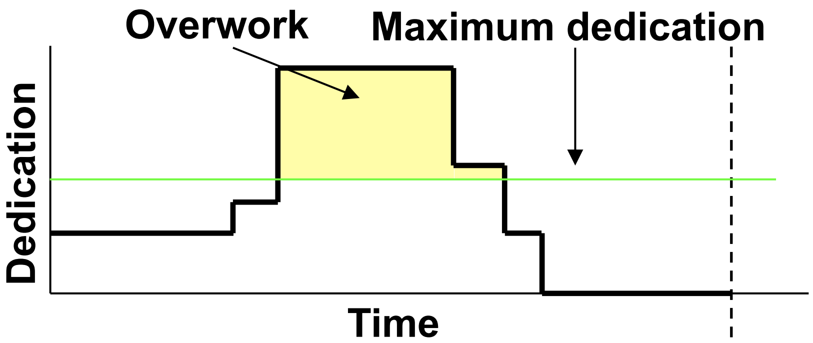
```
Objective function $f(\mathbf{x})$,        $\mathbf{x} = (x_1, ..., x_d)^T$
Generate initial population of fireflies $\mathbf{x}_i$ $(i = 1, 2, ..., n)$
Light intensity $I_i$ at $\mathbf{x}_i$ is determined by $f(\mathbf{x}_i)$
Define light absorption coefficient $\gamma$
while ($t <$MaxGeneration)
for $i = 1 : n$ all $n$ fireflies
   for $j = 1 : i$ all $n$ fireflies
         if $(I_j > I_i)$, Move firefly $i$ towards $j$ in d-dimension; end if
         Attractiveness varies with distance $r$ via $\exp[-\gamma r]$
         Evaluate new solutions and update light intensity
   end for $j$
end for $i$
Rank the fireflies and find the current best
end while
Postprocess results and visualization
```

$$\mathbf{x}_i = \mathbf{x}_i + \beta_0 e^{-\gamma r_{ij}^2}(\mathbf{x}_j - \mathbf{x}_i) + \alpha\,(\mathrm{rand} - \frac{1}{2})$$

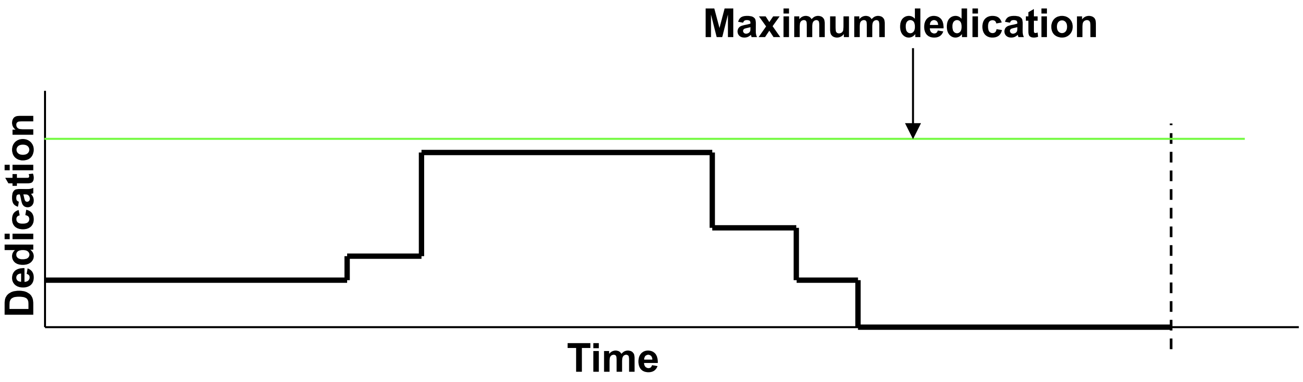
Algorithms: repair mechanism

- For constraint R3

$$\boxed{x'_{ij} = \frac{x_{ij}}{\max_{i,\tau}\{e_i^{work}(\tau)\} + \varepsilon}}$$

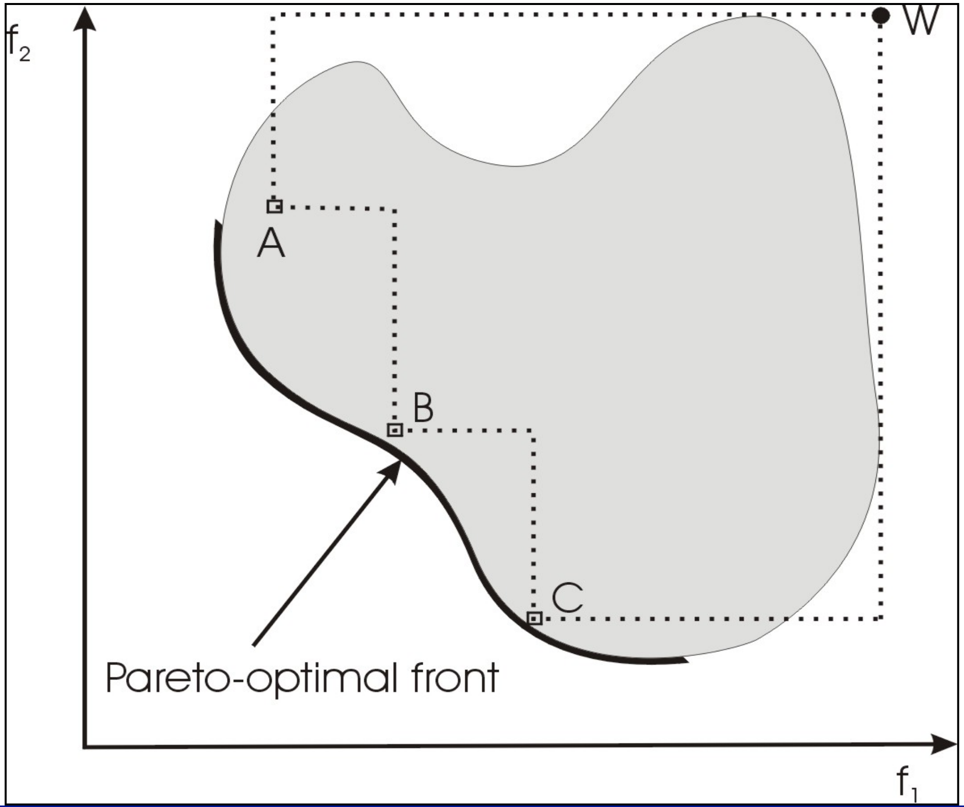


Repair mechanism



Experiments: Quality Indicators

- Hypervolume (HV)
  - Volume covered by members of the non-dominated set of solutions
  - Measures both convergence and diversity in the Pareto front
  - Larger values are better



- Attainment surfaces
  - Localization statistics for fronts
  - The same as the median and the interquartile range in the mono-objective case

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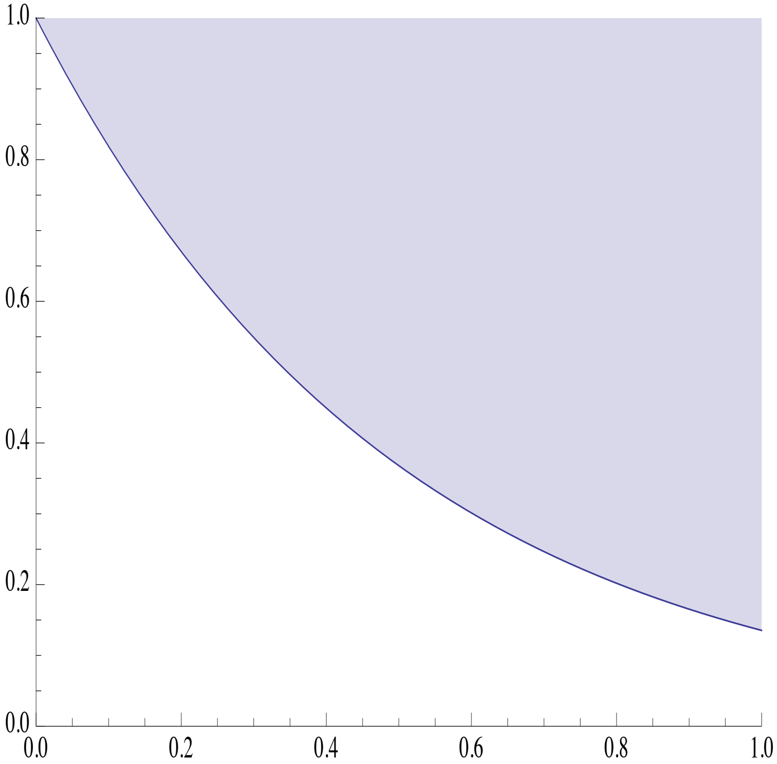

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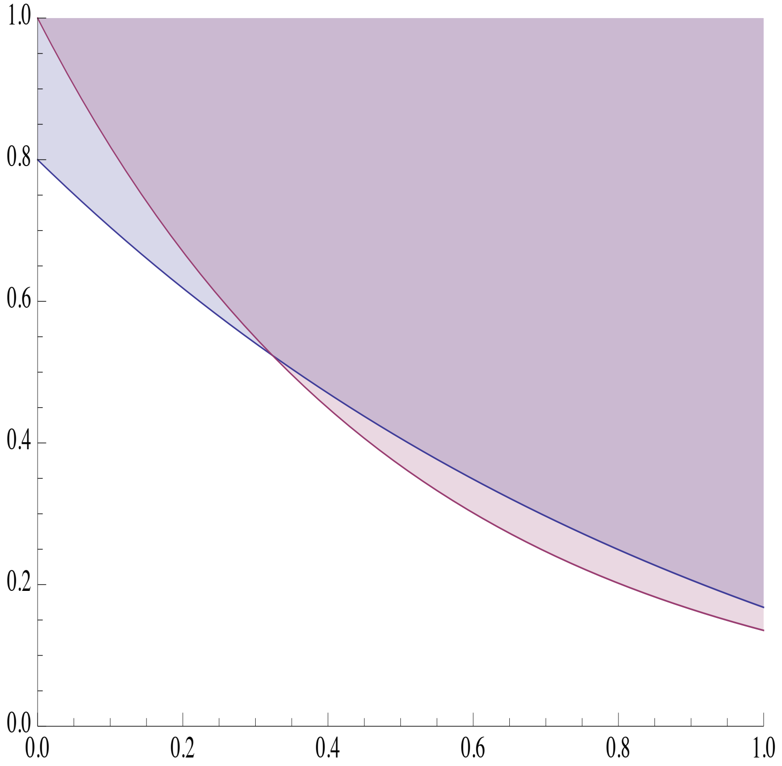

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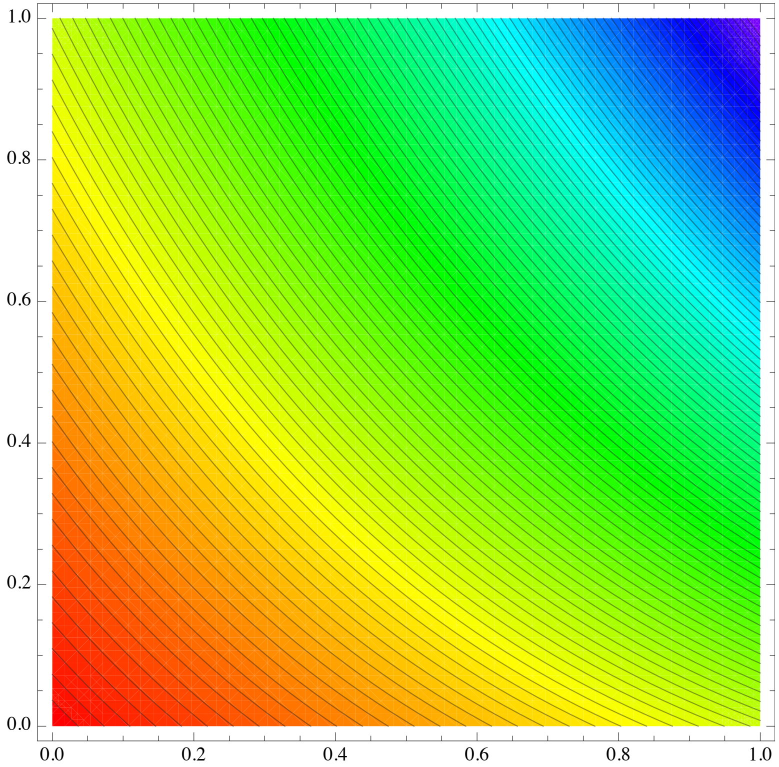

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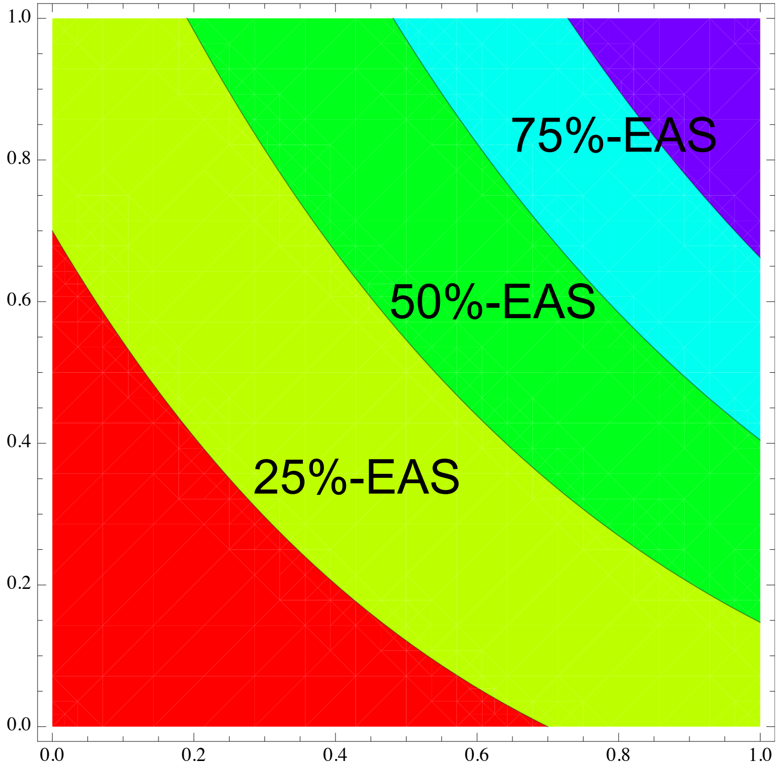


Experiments: Instances and Parameters

Problem instances

- 36 instances of increasing number of tasks (16-512) and employees (8-256)
- Labeled as i<tasks>-<employees>

Global Parameters

- Stopping condition: 100 000 function evaluations
- Approximated Pareto front size: 100 solutions
- 100 independent runs for each algorithm-instance
- Statistical tests for significance differences
- Representation: vector of real numbers

Experiments: Algorithm-Specific Parameters

NSGAII

- Population: 100
- SBX (η_c=20, p_c=0.9)
- Polynomial mutation (η_m=20, p_m=1/L)

PAES

- Population: 1
- Polynomial mutation (η_m=20)

DEPT

- Population: 32
- RandToBest/1/Binomial
- CR=0.9, F=0.5

MO-FA

- Population: 32
- Mutation factor: 0.5

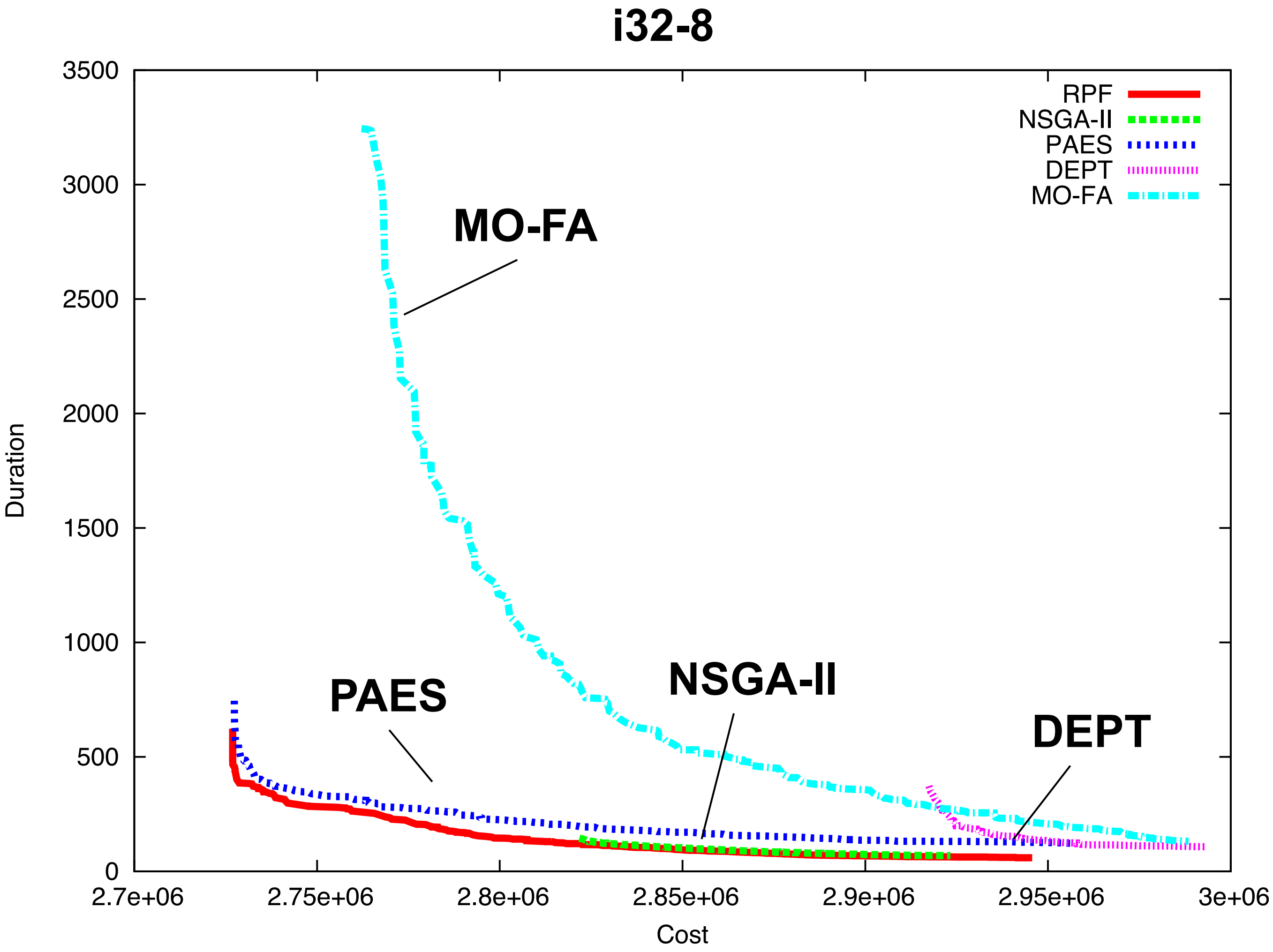
Results: Hypervolume Comparison

Hypervolume (HV)

- PAES is the clear winner in HV
- MO-FA is the second best for small instances
- NSGA-II is the second best for large instances
- DEPT is the worst algorithm in the comparison

	NSGA-II	PAES	DEPT	MO-FA
i16-8	$0.661_{\pm 0.028}$	$0.732_{\pm 0.019}$	$0.311_{\pm 0.020}$	$0.540_{\pm 0.022}$
i16-16	$0.468_{\pm 0.026}$	$0.826_{\pm 0.013}$	$0.327_{\pm 0.038}$	$0.608_{\pm 0.031}$
i16-32	$0.147_{\pm 0.016}$	$0.809_{\pm 0.009}$	$0.226_{\pm 0.037}$	$0.379_{\pm 0.088}$
i16-64	$0.129_{\pm 0.017}$	$0.858_{\pm 0.010}$	$0.287_{\pm 0.034}$	$0.370_{\pm 0.092}$
i16-128	$0.048_{\pm 0.013}$	$0.722_{\pm 0.012}$	$0.122_{\pm 0.028}$	$0.139_{\pm 0.075}$
i16-256	$0.018_{\pm 0.008}$	$0.682_{\pm 0.010}$	$0.078_{\pm 0.029}$	$0.069_{\pm 0.029}$
i32-8	$0.538_{\pm 0.029}$	$0.721_{\pm 0.017}$	$0.107_{\pm 0.024}$	$0.209_{\pm 0.029}$
i32-16	$0.190_{\pm 0.026}$	$0.820_{\pm 0.012}$	$0.035_{\pm 0.017}$	$0.341_{\pm 0.045}$
i32-32	$0.121_{\pm 0.014}$	$0.743_{\pm 0.009}$	$0.109_{\pm 0.018}$	$0.284_{\pm 0.042}$
i32-64	$0.049_{\pm 0.010}$	$0.795_{\pm 0.025}$	$0.042_{\pm 0.018}$	$0.263_{\pm 0.083}$
i32-128	$0.041_{\pm 0.007}$	$0.726_{\pm 0.012}$	$0.080_{\pm 0.015}$	$0.061_{\pm 0.011}$
i32-256	$0.009_{\pm 0.007}$	$0.617_{\pm 0.016}$	$0.011_{\pm 0.011}$	$0.000_{\pm 0.000}$
i64-8	$0.463_{\pm 0.031}$	$0.813_{\pm 0.014}$	$0.089_{\pm 0.014}$	$0.140_{\pm 0.020}$
i64-16	$0.221_{\pm 0.022}$	$0.959_{\pm 0.011}$	$0.026_{\pm 0.007}$	$0.347_{\pm 0.031}$
i64-32	$0.063_{\pm 0.011}$	$0.798_{\pm 0.008}$	$0.001_{\pm 0.002}$	$0.321_{\pm 0.031}$
i64-64	$0.073_{\pm 0.012}$	$0.870_{\pm 0.006}$	$0.004_{\pm 0.005}$	$0.032_{\pm 0.023}$
i64-128	$0.027_{\pm 0.007}$	$0.738_{\pm 0.008}$	$0.006_{\pm 0.005}$	$0.002_{\pm 0.004}$
i64-256	$0.000_{\pm 0.000}$	$0.618_{\pm 0.013}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i128-8	$0.320_{\pm 0.019}$	$0.986_{\pm 0.006}$	$0.003_{\pm 0.006}$	$0.076_{\pm 0.031}$
i128-16	$0.253_{\pm 0.023}$	$0.988_{\pm 0.013}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.001}$
i128-32	$0.213_{\pm 0.016}$	$0.980_{\pm 0.010}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i128-64	$0.095_{\pm 0.008}$	$0.920_{\pm 0.021}$	$0.000_{\pm 0.000}$	$0.044_{\pm 0.023}$
i128-128	$0.029_{\pm 0.008}$	$0.782_{\pm 0.009}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i128-256	$0.008_{\pm 0.005}$	$0.514_{\pm 0.017}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i256-8	$0.309_{\pm 0.015}$	$0.998_{\pm 0.002}$	$0.000_{\pm 0.000}$	$0.010_{\pm 0.019}$
i256-16	$0.156_{\pm 0.012}$	$0.987_{\pm 0.007}$	$0.000_{\pm 0.001}$	$0.281_{\pm 0.018}$
i256-32	$0.089_{\pm 0.009}$	$0.927_{\pm 0.012}$	$0.000_{\pm 0.000}$	$0.009_{\pm 0.012}$
i256-64	$0.028_{\pm 0.005}$	$0.739_{\pm 0.022}$	$0.000_{\pm 0.000}$	$0.002_{\pm 0.004}$
i256-128	$0.019_{\pm 0.005}$	$0.752_{\pm 0.010}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i256-256	$0.003_{\pm 0.003}$	$0.626_{\pm 0.016}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i512-8	$0.235_{\pm 0.016}$	$0.995_{\pm 0.002}$	$0.000_{\pm 0.000}$	$0.029_{\pm 0.025}$
i512-16	$0.103_{\pm 0.007}$	$0.960_{\pm 0.011}$	$0.000_{\pm 0.000}$	$0.025_{\pm 0.024}$
i512-32	$0.029_{\pm 0.008}$	$0.961_{\pm 0.010}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i512-64	$0.018_{\pm 0.007}$	$0.820_{\pm 0.014}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$
i512-128	$0.198_{\pm 0.004}$	$0.820_{\pm 0.010}$	$0.102_{\pm 0.003}$	$0.105_{\pm 0.006}$
i512-256	$0.008_{\pm 0.005}$	$0.560_{\pm 0.031}$	$0.000_{\pm 0.000}$	$0.000_{\pm 0.000}$

Results: 50%-Empirical Attainment Surfaces (EAS)



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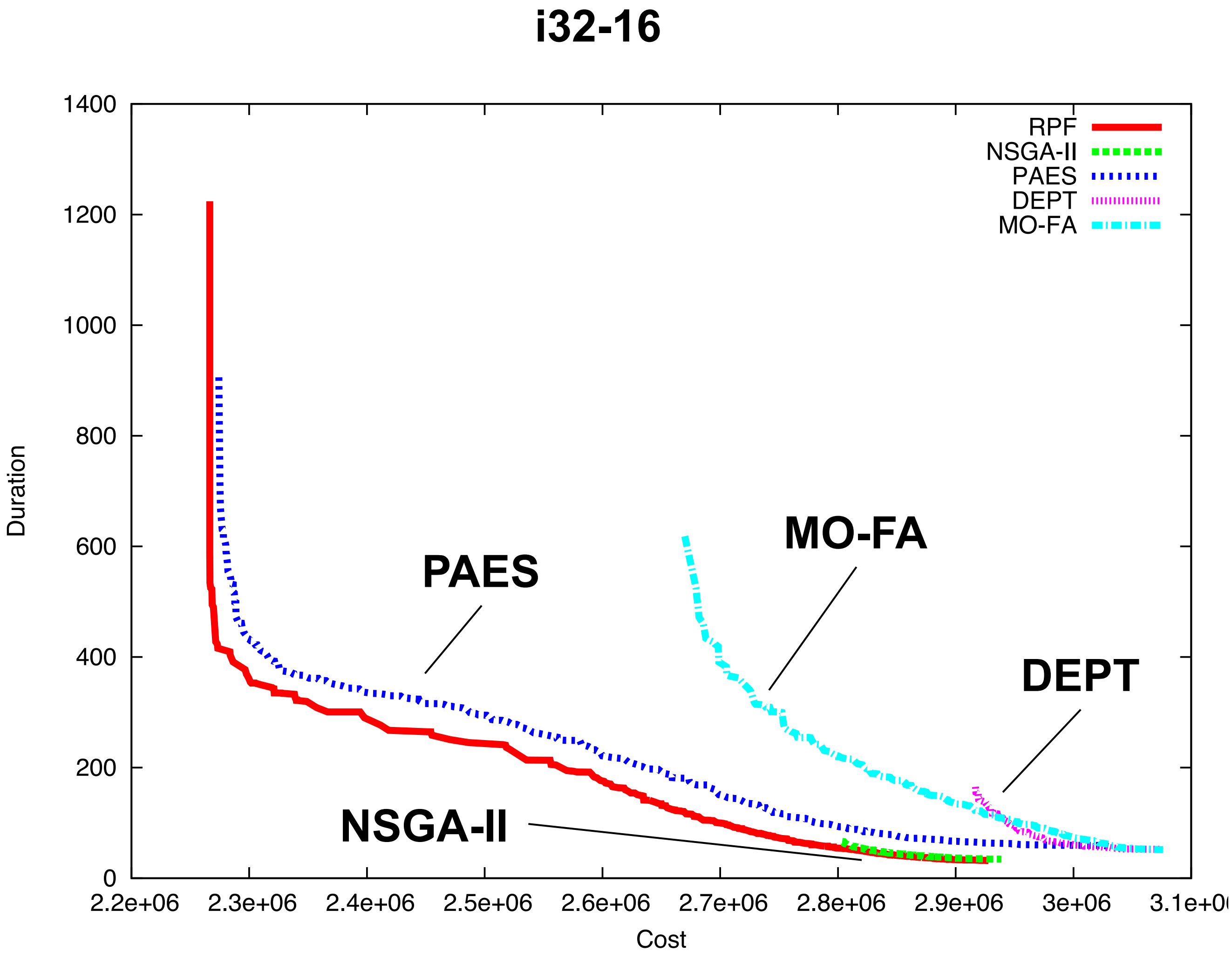

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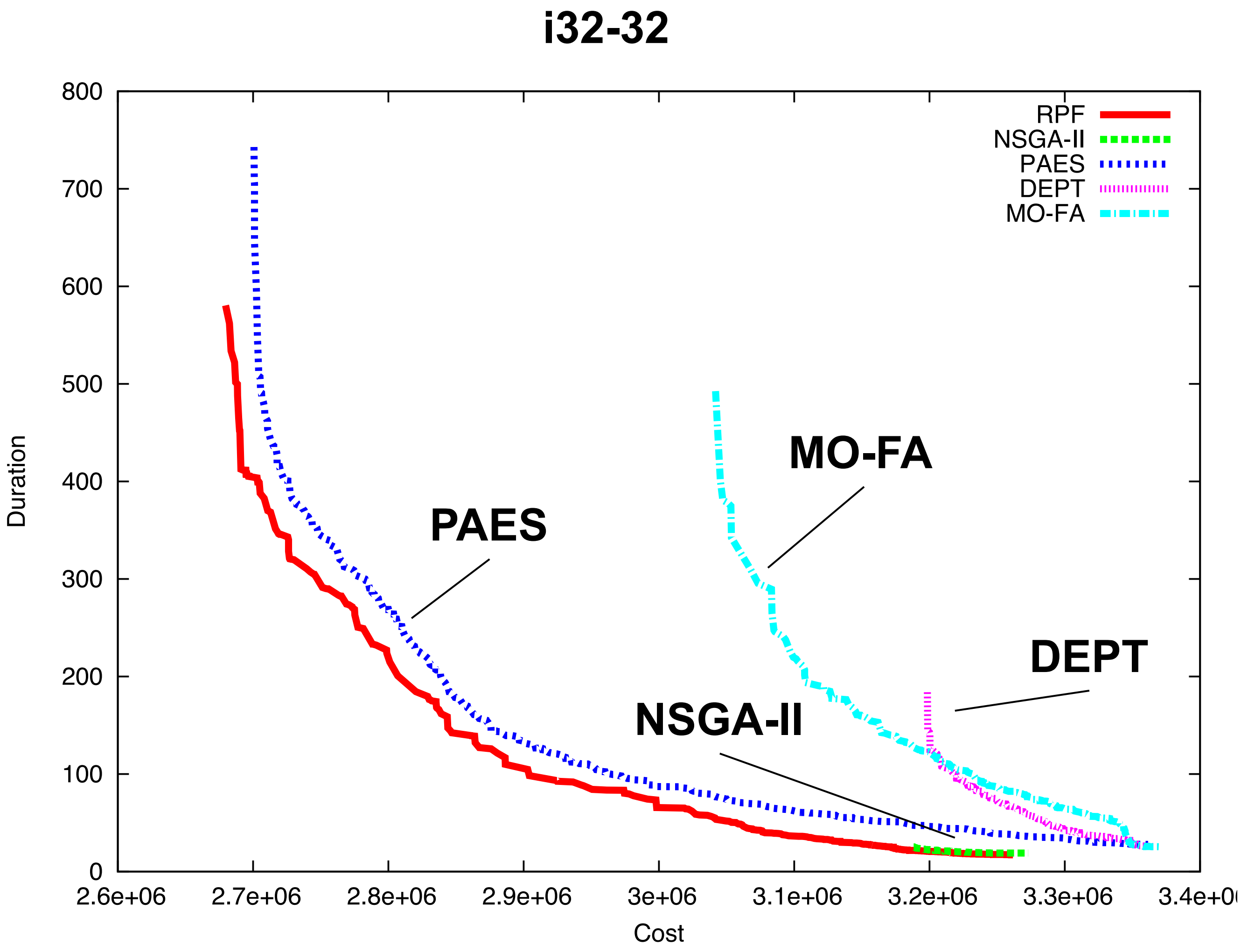

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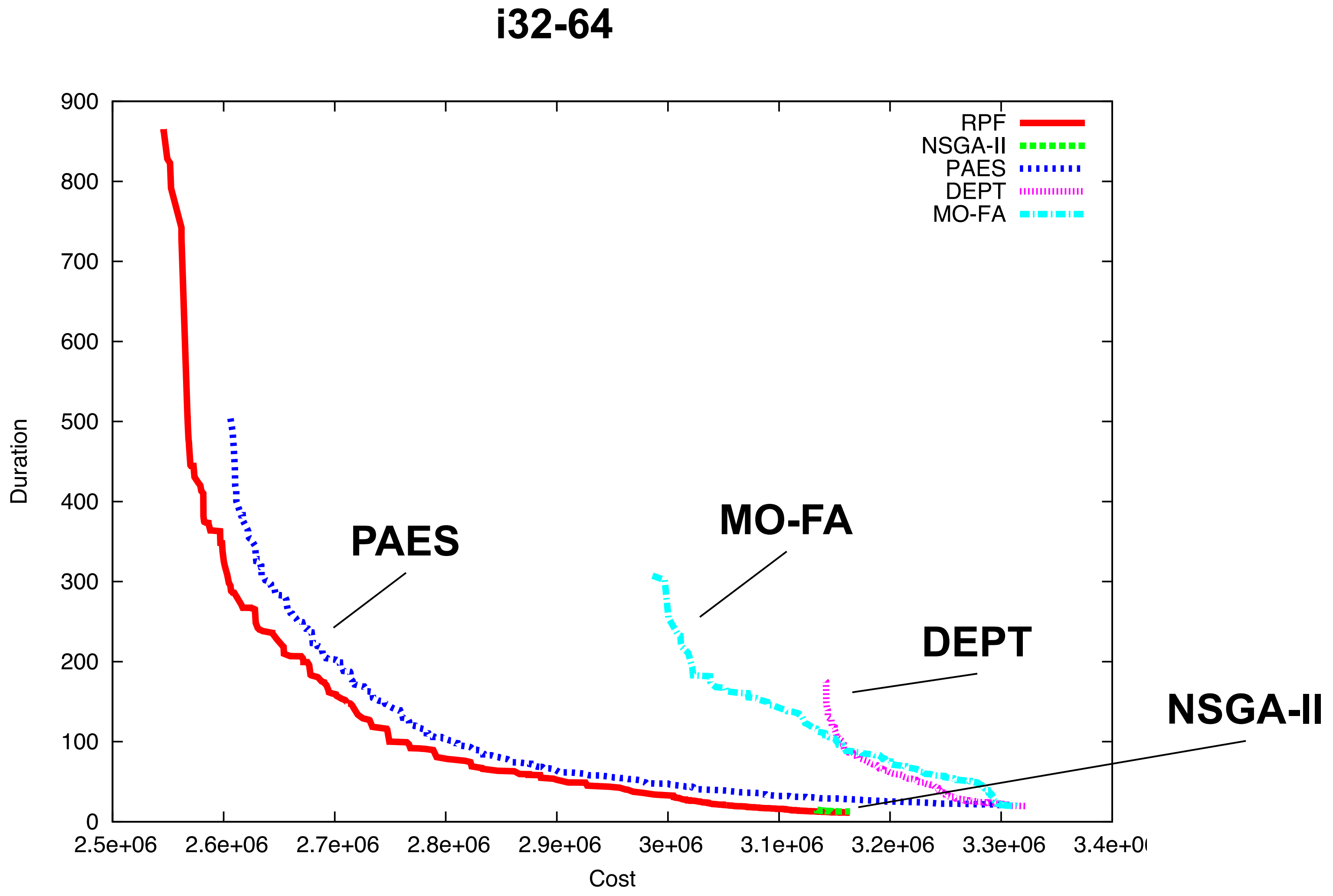

Conclusions & Future Work

Conclusions

- PAES is the algorithm with better scalability behaviour
- MO-FA is the second best in small instances
- NSGA-II is the second best in large instances
- DEPT is the worst algorithm in the comparison
- HV not always provides enough information to determine the best algorithm. EAS is a good alternative.

Future Work

- Use real-world instances of the problem
- Change the formulation of the problem to get closer to reality

Thanks for your attention !!!

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