Optimal Placement of Antennae using Metaheuristics

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Abstract. In this article we solve the radio network design problem (RND). This NP-hard combinatorial problem consist of determining a set of locations for placing radio antennae in a geographical area in order to offer high radio coverage using the smallest number of antennae. This problem is originally found in mobile telecommunications (such as mobile telephony), and is also relevant in the rising area of sensor networks. In this work we propose an evolutionary algorithm called CHC as the state of the art technique for solving RND problems and determine its expected performance for different instances of the RND problem.

1 Introduction

An important symbol of our present information society are telecommunications. With a rapidly growing number of user services, telecommunications is a field in which many open research lines are challenging the research community. Many of the problems found in this area can be formulated as optimization tasks. Some examples are assigning frequencies to cells in mobile communication systems [1], building multicast routing trees for alternate path computation in large networks [2], developing error correcting codes for the transmission of messages [3], and designing the telecommunication network [4, 5]. The problem tackled in this paper belongs to this last broad class of network design tasks. When a geographically dispersed set of terminals needs to be covered by transmission antennae a key issue is to minimize the number and locations of these antennae and cover a large area at the same time. This is the central idea of the *radio network design* problem (RND).

In order to solve RND, metaheuristic techniques are used to overcome the large dimension and complexity of the problem, often unaffordable for exact algorithms. In the associated literature the problem has been solved with genetic algorithms [6, 7]. In this article, our goal is to improve existing results and propose a state-of-the-art optimization method to solve the RND problem. In particular, we will compare the CHC algorithm against three other techniques: a simulated annealing (SA), a steady state genetic algorithm (ssGA), and a generational genetic algorithm (genGA). Another objective of this work is to extend the basic formulation of the problem to include more realistic kinds of antenna.

In summary, the contribution of this paper consists of the application of a an algorithm not previously used, CHC, that improves all the results in the literature, the optimization of the algorithm parameters, the analysis of the scaling properties of the RND problem, and the extension of the basic problem to include more than one type of antenna.

The paper is organized as follows. In the next section we define and characterize the radio network design problem. Section 3 briefly describes the CHC algorithm. Section 4 provides the results of the tests performed either to compare algorithms or analyze different types of antenna. Finally, some concluding remarks and future research lines are drawn in Section 5.

2 The Radio Network Design Problem

The radio coverage problem amounts to covering an area with a set of antennae. The part of an area that is covered by an antenna is called *a cell*. In the following we will assume that the cells and the area considered are discretized, that is, they can be described as a finite collection of geographical locations (taken from a geo-referenced grid).

Let us consider the set L of all potentially covered locations and the set M of all potential antenna locations. Let G be the graph, $(M \cup L, E)$, where E is a set of edges such that each antenna location is linked to the locations it covers and let the vector $\boldsymbol{x}$ be a solution to the problem where x_i with $i \in [1, |M|]$ indicates whether an antenna is being used or not at the ith available location.

Throughout this work we will consider different versions of the RND problem, which will differ in the type of antennae that might be placed in each location. There are simple versions using antennae that have no parameters, and more complex versions where antennae have parameters (i.e. azimuth) that determine the area they cover. In the last case, any solution $\boldsymbol{x}$ must also indicate which values the parameters of the antennae have for each antenna used.

Searching for the minimum subset of antennae that covers a maximum surface of an area comes to searching for a subset $M' \subseteq M$ such that $|M'|$ is minimum and such that $|Neighbors(M', E)|$ is maximum, where

$$Neighbors(M', E) = \{u \in L \mid \exists v \in M', (u, v) \in E\} \ . \tag{1}$$

The problem we consider recalls the Unicost Set Covering Problem (USCP) that is known to be NP-hard. An objective function to combine the two goals has been proposed in [6]:

$$f(\boldsymbol{x}) = \frac{Coverage(\boldsymbol{x})^{\alpha}}{Nb.\ of\ antennae(\boldsymbol{x})} \ , \ Coverage(\boldsymbol{x}) = \frac{100 \cdot Neighbors(M', E)}{Neighbors(M, E)} \ , \tag{2}$$

where the parameter α can be tuned to favor the cover rate factor with respect to the number of antennae. Just like Calégari et al. did [6], we will use $\alpha = 2$, and a 287×287 point grid representing an open-air flat area.

Three different antenna types will be used in this work: a square shaped cell antenna that covers a 41×41 point cell as used in [6, 7], an omnidirectional

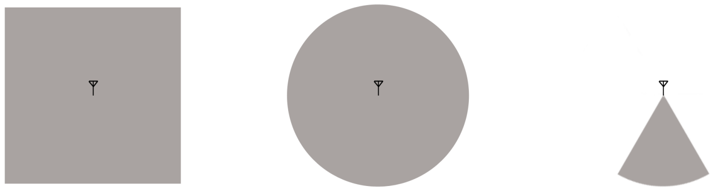

Fig. 1. Terrain coverages with different types of antenna

antenna that covers a 22 point radius circular cell (new contribution here), and a directive antenna that covers one sixth of the omnidirectional cell (new contribution here). When directive antennae are employed, three of them are placed in the location site. Fig. 1 illustrates the terrain coverages obtained with the different kinds of antenna.

3 The CHC Algorithm

The algorithm we propose for solving the RND problem is Eshelman's CHC, a kind of Evolutionary Algorithm (EA) surprisingly not used in many studies despite it has unique operations usually leading to very efficient and accurate results [8]. Like all EAs, it works with a set of solutions (*population*) at any time. The algorithm works iteratively, producing new solutions at each iteration, some of which will be placed into the population instead of others that were previously included. The pseudocode for this algorithm is shown in Fig. 2.

The algorithm CHC works with a population of individuals (solutions) that we will refer to as P_a. In every step, a new set of solutions is produced by selecting pairs of solutions from the population (the parents) and recombining them. This selection is made in such a way that individuals that are too similar can not mate each other, and recombination is made using a special procedure known as HUX. This procedure copies first the common information for both parents into both offspring, then translates half the diverging information from each parent to each of the offspring. This is done in order to preserve the maximum amount of diversity in the population, as no new diversity is introduced during the iteration (there is no mutation operator). The next population is formed by selecting the best individuals among the old population and the new set of solutions (elitist criterion).

As a result of this, at some point of the execution population convergence is achieved, so the normal behavior of the algorithm should be to stall on it. A special mechanism is used to generate new diversity when this happens: the *restart* mechanism. When restarting, all of the solutions except the very best ones are significantly modified (*cataclysmically*). This way, the best results of the previous phase of evolution are maintained and the algorithm can proceed again.

```
t:=0;
Initialize(Pa,convergence_count);
while not ending_condition(t,Pa) do
    Parents := Selection_parents(Pa);
    Offspring := HUX(Parents);
    Evaluate(Offspring);
    Pn := Elitist_selection(Offspring,Pa);
    if not modified(Pa,Pn) then
        convergence_count := convergence_count-1;
        if (convergence_count == 0) then
            Pn := Restart(Pa);
            Initialize(convergence_count);
        end if
    end if
    t := t+1;
    Pa := Pn;
end while
```

Fig. 2. Pseudocode for CHC

4 Experiments

In this section we briefly present the results of performing an assorted set of experiments to solve the different RND problems using CHC. First we solve RND problems where antennae have no parameters. In this part, CHC will be faced against three other algorithms: SA, ssGA, and genGA, and the results will be compared to the best results of the literature [7] (dssGA8). Afterwards, we tackle the problem using antennae with parameters that shape the coverage cell. Only CHC will be employed in this part. Its behavior when facing different problem types will be studied here.

For each experiment, we will analyze the number of evaluations required to solve the problem if the execution is performed until an optimal solution is found (whenever possible). We perform 50 independent runs of each experiment. A statistical analysis is driven to validate the results obtained during the tests. The values of the parameters employed for CHC are shown in Table 1. When a range of values is shown instead of a single value, it means either that the parameter is tuned (population size) or that the value is selected to be adequate for each problem instance (maximum evaluations).

Table 1. Parameters of the CHC algorithm

Maximum evaluations	2,500,000−50,000,000
Crossover probability	0.8
Restarting mutation probability(%)	35
Size of population	50−10,000

4.1 RND with Squared and Circular Cell Antennae

In squared and circular cell antennae instances a solution is encoded with a bit string, where each bit relates to an available location site and determines whether an antenna is placed there (1) or not (0). Let L be the problem size (the number of available location sites), the size of the solution space for these instances is 2^L. For each instance the optimal solution is known beforehand.

The scalability of the problem is also studied by solving instances of sizes ranging from 149 to 349 available locations. Every time an algorithm is applied to solve an instance, we perform a parameter tuning in order to obtain the best possible performance from that algorithm. For the CHC algorithm the parameter tuned is the population size.

The results of the experiments are shown in Table 2 for square shaped cell antennae and Table 3 for omnidirectional antennae. All the algorithms were able to solve the problem with very high hit ratio (percentage of executions where the optimal solution is found) except a few exceptions (highlighted in italics), therefore only the number of evaluations is shown. The best results obtained are highlighted in in boldface. A Student t-test shows that all differences between CHC and the rest of algorithms are statistically significant with 95% of confidence.

Table 2. Comparison of the number of evaluations required by the different algorithms in RND with square shaped coverage antennae

Algorithm	**Size**				
	149	**199**	**249**	**299**	**349**
CHC	**30,319**	**78,624**	**148,595**	**228,851**	**380,183**
SA	86,761	196,961	334,087	637,954	810,755
ssGA	239,305	519,518	978,573	1,872,463	3,460,110
genGA	141,946	410,531	987,074	1,891,768	3,611,802
dssGA8 [7]	785,893	1,467,050	2,480,883	2,997,987	4,710,304

Table 3. Comparison of the number of evaluations required by the different algorithms in RND with omnidirectional antennae

Algorithm	**Size**				
	149	**199**	**249**	**299**	**349**
CHC	**45,163**	**344,343**	**817,038**	**2,055,358**	**3,532,316**
SA	83,175	*262,282*	913,642	2,945,626	*6,136,288*
ssGA	365,186	1,322,388	2,878,931	*9,369,809*	*9,556,983*
gGA	206,581	1,151,825	3,353,641	*8,080,804*	*19,990,340*

CHC proves to be the best technique among the four: it gets the lowest solving costs for all instances. In the first case (square shaped coverage) it improves the second best technique, SA, by costing less than 50%. In the second case (omnidirectional), the cost reduction regarding the second best technique (SA) is comprised between 10% and 40% (in the 199-size instance SA has a lower solving cost, but gets a low hit ratio). In both cases the increase of the number of evaluations is clearly superlineal, however, numeric approximations have returned subexponential models.

If we compare the two variants of RND (differing on the kind of antenna employed), we observe that the one using omnidirectional antennae seems to be more difficult to solve, since for the same instance size the required number of evaluations is higher. Furthermore, the problem becomes less tractable when its size grows, and the gap between efforts for solving the two kinds of problem increases.

In summary, CHC is better suited for solving RND than SA or any of the GAs. It is the best for the basic instance and allows a better scalability than the other two. The change of the antenna cell shape modifies the complexity of the optimization problem, but does not change the fact that the best results are obtained with the CHC algorithm. Therefore, from this point we will only employ CHC to solve the new instances of RND.

4.2 Complex RND Variants

Two variants of the RND are solved in this section: RND using directive antennae and RND using all kinds of antenna. When directive antennae are used, either three of them or none are placed in each available location. When three of them are placed, they are subject to one of the following restrictions: all antennae of the same location site must point in consecutive directions (case 1) or in different directions (case 2). When all kinds of antenna are employed, the restriction over the directive antennae is the second one (case 2).

The number of available locations of the instances considered is limited in both cases to only 149 as a base line for future research. For practical means, we will use the binary equivalent length (minimum length of a binary string that can store all the possible values of the solution space) as the instance size measure. Table 4 shows CHC's performance for all the problem instances solved in this work.

Table 4. Comparison of CHC's best performances for all the problem instances

Problem Instance		**Binary Size**	**Fitness Evaluations**	**Optimal Population**	**Running Time(sec)**	**Hit Rate(%)**
Square	149	149	30,319	400	25.59	100
	199	199	78,624	1,200	76.66	100
	249	249	148,595	1,400	146.21	100
	299	299	228,851	1,800	237.38	100
	349	349	380,183	2,800	427.81	100
Omnidirectional	149	149	45,163	700	43.71	100
	199	199	344,343	2,800	374.01	100
	249	249	817,038	4,000	870.82	100
	299	299	2,055,358	8,000	2437.51	100
	349	349	3,532,316	10,000	4009.85	100
Directive	case 1	419	2,383,757	4,000	4186.38	96
	case 2	655	4,736,637	8,000	9827.60	88
All antennae		675	829,333	10,000	1284.05	100

Fig. 3 illustrates the cost and size of all the different problem instances solved in this work: those using squared cells (unlabelled squared points), those using circular cells (unlabelled circular points), the ones using directional antennae under the first restriction (RND-3) and the second restriction (RND-4), and the variant using all antenna kinds (RND-5). Minimal mean square error approximations for the problems using squared cells and circular cells are also shown.

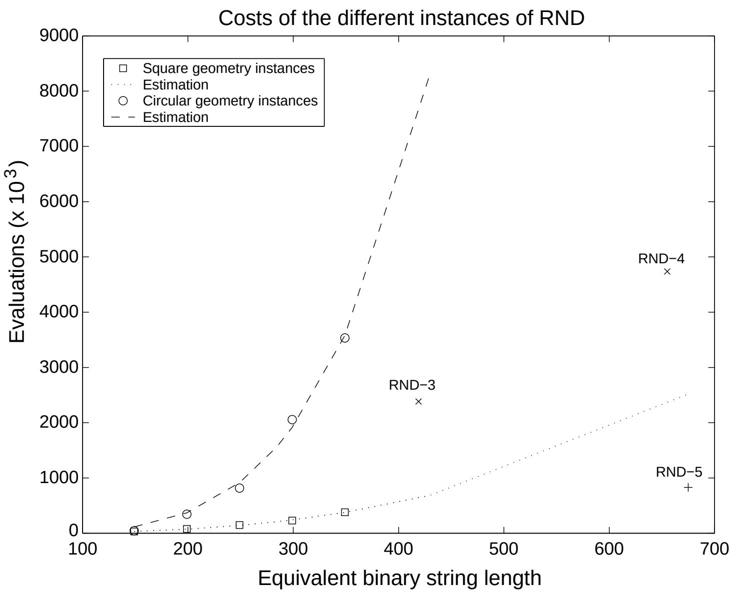

Fig. 3. Comparison of the evaluations performed by CHC for several problem instances

The problem variant using directive antennae seems to have a cost-size relation comprised between those of the variants using squared cell antennae and omnidirectional antennae. However, problem instances using only directive antennae do not have one single optimal solution (as the previous variants do), but a set of optimal solutions instead: 6^{52} and 20^{52} for the instances under the first and the second restriction respectively. Therefore the complexity reduction of this RND variant regarding the omnidirectional antennae variant might be due to the existence of many optimal solutions.

The variant of the problem using all antenna kinds simultaneously seems to have a cost-size relation lower than any of the other variants: for a binary length of 675 (93% higher than the 349 squared coverage instance) its solving cost is only $829,333$ (118% higher). This would approximately correspond to a lineal growth, yet the measured growth has been estimated to be superlineal.

Therefore, the studied RND problems can be classified into two main different categories depending on their cost-size relation: a low complexity kind (x^3 law), and a high complexity kind (x^4 law). The geometry of the cell shape seems to be the decisive factor: both directive and omnidirectional antennae share the circular geometry so the two belong to the high complexity kind. The square shaped cells problem variant belongs to the low complexity kind. The variant of the problem where all antenna kinds are used simultaneously takes advantage of the possibility of using both geometries and achieves a complexity lower than any of the other variants.

5 Conclusions

We have established CHC as the best technique so far for solving the RND problem. This has been proven empirically by comparison with SA, ssGA, and genGA in two different scenarios: use of square shaped cell antennae and use of omnidirectional antennae. The cost of solving the problem has been estimated to grow in a subexponential manner as the size of the problem increases. The nature of that increase is mainly determined by the geometrical features of the antennae, being x^3 for square shaped cell and x^4 for circular shaped cell antennae. When directive antennae are placed, the fact of having many optimal solutions results in a cost reduction with respect to the RND using omnidirectional antennae. When several antennae are offered, the algorithm takes advantage of it and is able to solve the problem at lower cost.

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